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Research paper

A new fuzzy framework for the optimal placement of phasor measurement units under normal and abnormal conditions

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ABSTRACT

This paper presents a new procedure for finding the optimal placement of the phasor measurement units (PMUs) in modern power grids to achieve full network observability under normal operating conditions, and also abnormal operating conditions such as a single line or PMU outage, while considering the availability of PMU measuring channels. The proposed modeling framework is implemented using the fuzzy binary linear programming (FBLP) technique. Linear fuzzy models are proposed for the objective function and constraints alike. The proposed procedure is applied to five benchmark systems such as the IEEE 14-bus, 30-bus, 39-bus, 57-bus, and 118-bus. The numerical results demonstrate that the proposed framework is capable of finding a fine-tuned optimal solution with a simple model and acceptable solution characteristics compared with early works in the literature. Besides, four evaluation indices are introduced to assure the various criteria under study such as the observability depth, measurement redundancy, and robustness of the method under contingencies. The results show that full network observability can be met under normal conditions using 20% PMUs penetration; however, under contingencies, approximately 50% PMUs penetration is required. The novelty of the proposed procedure has proven the capability of the proposed linear fuzzy models to find better optimal number of PMUs with lower number of channels compared to other algorithms under various operating conditions. Hence, the proposed work represents a potential tool to monitor power systems, and it will help the operators in a smart grid environment.

Introduction

Smart grids (SGs) benefit from the continuous improvement in power systems control and the advances in the intelligent measurement technologies. However, controlling electric power systems is becoming more and more cumbersome because of the development of power grid structure and the advance in power markets complexity, especially in deregulated electricity markets. Accordingly, efficient utilization of energy resources is a crucial requirement for incorporating SGs that can present a highly reliable power system with the optimal use of all the available resources. The existing power grids all over the world require revolutionary changes to meet the dramatic growing demands and also to make the grid smarter and more reliable. Also, the instantaneous monitoring of the voltage, current and injected power at all buses in the network is another essential requirement for the SGs as the traditional monitoring systems cannot satisfy this requirement since they are designed based on nonlinear power flow equations [1,2].

Synchronized phasor measurement devices like PMUs, which were first launched in 1980, can measure values of phasor voltages at the buses where they are located. The phase angles of the bus voltages measured in the real-time domain have not been possible yet, as the synchronizing measurements from remote locations are technically challenging. PMUs can alleviate this problem by realizing a synchronization for voltage and current waveforms at remote locations using the global positioning system (GPS) clock, which has an accuracy less than 1 μs; this enables new avenues in power systems monitoring, security analysis, protection and stability control. One such new application of PMUs in power systems is the fault location detection [1].

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PMUs present a real-time snapshot of the entire power system with several critical benefits over the conventional measurement systems, as detailed in [3], such as the high sampling rate to enhance the power system dynamic behavior, typically above 30 measurements every second, while providing more simple linear state estimation compared to the conventional nonlinear state estimation.

PMU placement at all buses would permit direct measurement of the network state. But equipping the power system with PMUs at 100% of the buses is not practical or reasonable because of high costs of the devices, and also limitations of the substation communications network. However, complete observability is very desirable in power systems; therefore, the problem of placing PMUs in a power system to attain a full system observability can be formulated as a constrained optimization problem [4–8]. However, if the lowest number of PMUs is used, there can be situations when limited communication channels or PMU outages could lead to buses becoming unobservable. Therefore, these contingencies have to be taken into account during the design stage.

In the literature, several studies have dealt with it. However, most of them intended to minimize the number of installed PMUs using different sets of constraints to attaining a complete topological observable system. In the studies presented by [9–12], the optimal PMU placement under full observability constraint is given. The availability of communication channels was tackled in [13–15]. In [15], the Markov process was employed to find the optimal solution considering channel limitations. However, the channel availability at each PMU is not considered, nor the single-line or PMU contingencies. In [16], a cellular learning automata (CLA) method is proposed to find the optimal solution for a complete system observability. However, the number of PMU measuring channels was not considered. In [17], a heuristic method was developed for simultaneous optimal PMU placement and phasor data concentrators (PDCs) in a hierarchical structured wide area monitoring system (WAMS). In [18], the authors introduced three approaches to determine the optimal solution for complete observability, namely depth first search (DFS), simulated annealing (SA), and minimum spanning tree (MST). The authors presented a three-stage method that found the optimal solution using network connectivity information in [19]. In the first two stages, PMUs are placed initially at all buses, while in the second stage, the set of connected PMUs was pruned to obtain the optimal PMUs locations. A heuristic search method was proposed for complete topological observability in [20] for full numerical observability in [21]. In [22], the grenade explosion method (GEM) was proposed for finding the optimal solution to provide complete observability. Besides, different meta-heuristic techniques have been suggested for solving the optimal PMU placement problem such as the simplified chemical reaction optimization (SCRO) [23], cellular genetic algorithm (CGA) [24], hybrid discrete particle swarm optimization (HDPSO) [25], particle swarm optimization algorithm [26,27], non-dominated sorting differential evolution (NSDE) algorithm [28], and topology based formulated algorithms and branch and bound (B and B) optimization technique [29]. Also, a method to approach the optimal PMU placement problem with random component outages (RCOs) was suggested in [3]. In the RCO's model, the state estimation error covariance is minimized. In [30], large-scale power networks were considered when formulating the optimal placement of monitoring devices for fault location. Several methods were further presented in [31–36], which involve the ant colony and integer linear programming optimization techniques.

Fuzzy logic is one of the efficient tools that incorporated in the field of power systems. Several applications of fuzzy logic are examined in [37,38]. In [39], the prioritization of different PMU placement configurations is based on multi-criteria decision-making schemes such as the analytic hierarchy process (AHP), or the simple weighted average method. A revised AHP for PMU allocation is presented in [6].

The traditional PMU allocation problem can be formulated as a binary linear programming (BLP) problem. However, conventional BLP problems are hard to solve due to uncertainties present in parameters and the structure; and this may lead to uncertainties in the decision space. On the other hand, the fuzzy set theory can be successfully applied under these uncertainties. Added to that, allocation of PMU devices faces the problem of uncertainty; this implies that reducing the number of PMU devices does not mean it reaches a unique solution. Moreover, it would be nonsensical to expect that the selected optimal set of PMUs will be installed all at once. The problem with PMU placement aims at minimizing the number of PMU placement as much as possible. In fact, the expression "as much as possible" conveys to the fuzziness in this problem. Consequently, the PMU placement problem can be modeled in fuzzy environment. In [40], the optimal PMU placement was approached using fuzzy weighted average. An approach for optimal placement of PMU devices considering fuzzy logic based critical buses was presented in [41]. The basics of the FBLP solution methodology were given in [42]. Other efforts to solve the PMU placement problem were presented in studies [43–49]. In the studies presented by [43,48], two optimal PMU location approaches were presented for power system state estimation. The search optimization techniques were developed in [44–48]. In [50], a Lyapunov exponent-based approach for optimal placement of PMUs was presented to attain full network observability, and to develop real-time system stability monitoring and assessment. The proposed optimal PMU placement method is limited and tested on the IEEE 39-bus test system only. Finally, given the fact that less attention was paid to the application of fuzzy logic to the optimal placement problem as noticed in the literature; accordingly, this paper proposes a novel linear fuzzy modeling of the allocation of PMUs problem to achieving complete system observability. A comparative study is presented to evaluate the proposed procedure for normal and abnormal operating conditions.

Material and methods

Problem formulation

A PMU device installed at bus '*i*' measures the voltage phasor of that bus (magnitude/angle) and the phasor currents (magnitude and angle) of the outgoing lines from the bus directly [43]. The number of measured current phasors depends on the availability of PMU channels. The optimal PMU placement problem for full observability considering PMU measuring channels can be expressed as an optimization problem, as follows:

$$\text{Minimize } f(x) = \sum_{i=1}^{n_b} \omega_i x_i \tag{1}$$

Subject to:

$$g(x) = \sum_{i,j}^{n_b} A_{ij} x_j > b_i \tag{2}$$

where, $f(x)$ is the objective function has to be minimized concerning the number and locations of PMUs with measuring channels. ω_i refers to the normalized weighting factor of the PMU located at bus '*i*', and this factor reflects the priority of each bus regarding the predetermined channel numbers at that bus. n_b is the size of the system, *i.e.* number of buses, and x_i is a vector represents the decision variables in a binary integer form.

$$x_i = \begin{cases} 1, & \text{if a PMU is installed at bus } i \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

This objective function represents the weighted sum of the employed PMUs. The normalized weighting factors are based on the system configuration which is exemplified by the total number of lines connected to bus i plus one that is equivalent to the number of PMU measuring channels. $g(x)$ given in Eq. (2) is the observability constraint that must be verified at each bus in the system, where b is the limiting boundary which refers to the observability requirement. Besides, the boundary limit 'b' indicates the measurement redundancy (MR) at each bus in the system. A_{ij} is the connectivity matrix which can be constructed based on the system line data in a binary integer form, so that

$$A_{ij} = \begin{cases} 1, & \text{if } i = j \\ 1, & \text{if buses } i \text{ and } j \text{ are connected} \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

The framework is intended for maintaining the system fully observable for all operating conditions, including emergency conditions such as single PMU contingencies. Each bus is observed by a directly connected PMU or by a pseudo-measurement; however a bus can be observed twice by one PMU, as the case of the double circuit. Redundancy is necessary for reliability; therefore at least two PMUs should observe each bus. Hence, a single PMU contingency would sustain observability. In other words, the framework aims at preserving the monitoring system as reliable as possible. To achieve the previous observability requirement; the limiting boundary b should be the minimum limit of MR that may be taken as;

$$b = \begin{cases} 1, & \text{for normal operating conditions} \\ 2, & \text{for any single line or PMU outage} \end{cases} \tag{5}$$

According to Eq. (5), a system is fully observable in normal operating conditions if b is equal to 1, which means that each bus shall be observed at least once. Regarding full observable system during contingencies such as any single line or PMU outage, each bus shall be observed at least twice by assigning b equals to 2. The PMU placement problem is to identify strategic buses, heavily loaded buses and the buses candidate for possible future expansion, at which PMUs will be installed.

The following observability rules are considered in the current allocation procedure [29].

Rule 1: If a bus is outfitted with a PMU, then the voltage and current phasors of all lines connected to that bus are considered known. This rule is satisfied with the proposed allocation procedure, *i.e.* with the normalized weighting factor given in (1).

Rule 2: If the current phasor is known on a line, and the voltage at one end of the same line is known then the voltage phasor at the other end of the line can be calculated. The observability constraints in (2) correctly reflect this rule. As one PMU, at certain bus i, is sufficient for full observability of system buses connected to it.

Rule 3: If the voltage phasors are measured at both ends of a line, then the current phasor on the same line can be calculated using the Kirchhoff Current Law (KCL).

Rule 4: For a zero-injection bus (ZIB) with no PMU installed, if the current phasors of all the incident lines are known, but one of the incident lines is unknown, then the unknown current phasor can be determined from KCL.

Rule 5: For a ZIB, if the voltage phasors of all adjacent buses are known, then the voltage phasor of the ZIB bus can be determined from the node voltage equations.

Rule 6: For a set of adjacent ZIBs, if the voltage phasors of the buses adjacent to them are known, then the voltage phasors of the initial set of adjacent ZIBs can be calculated from node voltage equations.

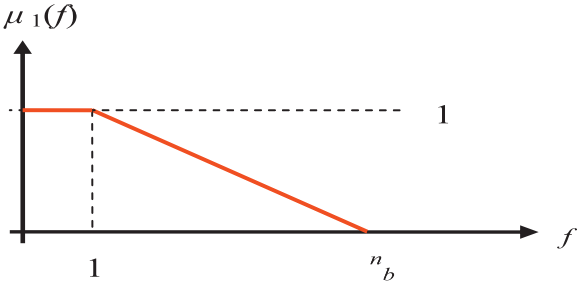

Fig. 1. Fuzzy membership model for the objective function.

It should be mentioned that **Rule 1** refers to so-called direct measurements, but **Rules 2** and **3** refer to the pseudo-measurement. The rest is conditioned to the zero injection buses (ZIB).

The fuzzy model of the objectives and constraints considers the previous six rules. The first rule has a direct effect on the normalized weighting factor in the problem objective function, while the observability constraints represent the other rules. Hence, in this work, the objective of the PMU placement problem is to minimize the PMUs number that can make the system fully observable and to maximize the measurement redundancy in the system. Therefore, for the position vector of each particle, the objective function should evaluate: (i) whether the system is observable, (ii) the number of PMUs employed in case the system is observable, and (iii) the measurement redundancy as defined as in [25,26,51].

Fuzzy modeling of the objective function and the constraints

In the literature, numerous types of fuzzy membership functions have been used in power system applications [37,38,40–42]. In this work, linear membership functions are suggested to characterize the control and dependent variables. The following subsections describe in details the fuzzy framework proposed for the objective function and the constraints at normal and abnormal operating conditions.

Fuzzy modeling of the objective function: The fuzzy number of the objective function of the planned PMU devices is inversely proportionally to the membership degree which equals 1 when a less number of PMUs is needed, but equals zero when a higher number of PMUs is required. Fig. 1 shows the fuzzy modeling of the objective function f.

It can be noted that a unity membership function corresponds to the lowest number PMU devices. Also, Eq. (6) gives the membership function of the objective function, $\mu_1(f)$.

$$\mu_1(f) = \frac{n_b - f}{n_b - 1}, \quad 1 \le f \le n_b \tag{6}$$

Substituting (1) into (6), then

$$\mu_1(f) = \frac{n_b - \left(\sum_{i=1}^{n_b} \omega_i x_i\right)}{n_b - 1}, \quad 1 \le f \le n_b \tag{7}$$

According to (7), the membership degree is dependent on the system size as well as the normalized weighted factor which reflects the priority and the number of channels needed for each bus.

Fuzzy modeling of the control variables: The fuzzy modeling for the PMU state at each bus, *i.e.* the control variable (x_i), is shown in Fig. 2.

The meaning behind assigning this member function is to allocate the least number of PMUs. Therefore, the membership degree must be assigned to 1 when the PMU is not allocated at bus i. The membership degree at bus i vanishes when the PMU device is allocated. The membership function for the control variables (x_i), is

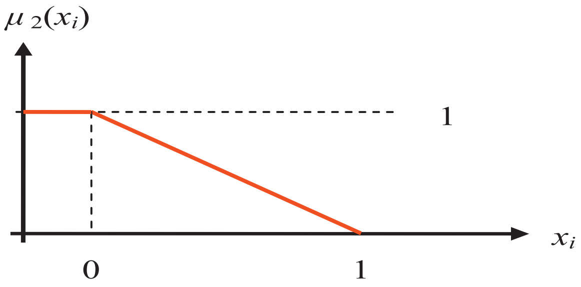

Fig. 2. Fuzzy membership model for the control variable x_i.

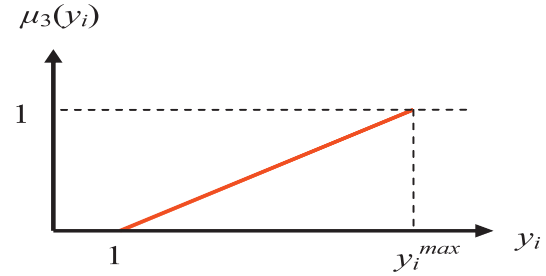

Fig. 3. Fuzzy modeling of observability constraints for normal operating condition.

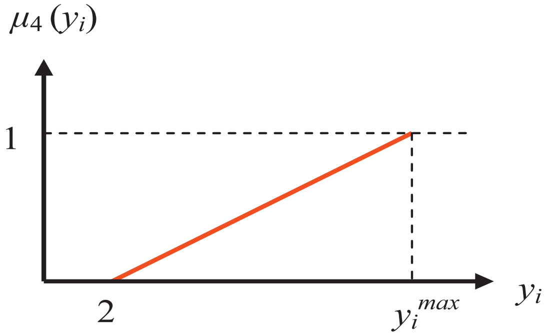

Fig. 4. Fuzzy modeling of observability constraints for single PMU/line outage.

given as

$$\mu_2(x_i) = -x_i + 1, \qquad 0 \le x_i \le 1 \tag{8}$$

Fuzzy modeling of the observability constraints: The fuzzy modeling of the observability constraints (previously defined in Eq. (2)), which reflect the observability rules 2–6, is shown in Fig. 3. The highest membership degree is assigned to the highest number of buses seen from bus i where the PMU device is installed. The membership function for the observability constraint ($\mathrm{y_i}$) at bus i, is described as;

$$\mu_3(y_i) = \frac{y_i - 1}{y_i^{\max} - 1}, \qquad 1 \le \mathrm{y_i} \le y_i^{\max} \tag{9}$$

Also, the fuzzy modeling of observability constraints for single PMU/line outage is given in Fig. 4. The membership function for the observability constraint (y_i) at bus i under the contingency condition is described as;

$$\mu_4(y_i) = \frac{y_i - 2}{y_i^{\max} - 2}, \qquad 2 \le \mathrm{y_i} \le y_i^{\max} \tag{10}$$

Additional rules of the proposed allocation procedure aim at further reduction of the number of PMU measuring channels while each bus is being observed at least once at normal operating conditions and twice at system contingencies are given below. The proposed reduction strategy (RS) rules are based on the MR of the system buses, the connectivity matrix A, and the status of system operating conditions.

1) If a bus k is observed n times so that $(n > 1)$ at normal operating conditions, then the reduction in PMU measuring channels which are responsible for this observability is $(n-1)$. Therefore, the bus k will be observed once from only one PMU. This rule is achieved as shown in Fig. 3.
2) If a bus l is observed m times so that $(m > 2)$ at system contingencies, then the reduction in PMU measuring channels which are responsible for this observability is $(m-2)$. Therefore, bus l will be observed twice from only two PMUs. This rule is achieved as shown in Fig. 4.

Hence, the previous fuzzy modeling considers these rules as presented in (9) and (10).

An additional important issue is the existence of the ZIB, which corresponds to transshipment nodes in the network. Excluding the ZIB from the control variables reduce the problem size. The fuzzy membership degree at the ZIBs equals 1 which means that no PMU is allocated at these buses.

The proposed solution procedure

The problem is formulated as BLP with fuzzy objectives and constraints, *i.e.* fuzzy binary linear programming (FBLP) which has the following form:

$$\text{Maximize} \sum_{i=1}^{n_b} \bar{c}_i x_i \tag{11}$$

Subject to:

$$\sum_{i=1}^{n_b} \bar{a}_{ij} x_j \le \bar{b}_j \qquad j = 1, 2, \ldots\ldots\ldots\ldots\ldots, m \tag{12}$$

where, x_i represents the status variable of the PMU device which expresses zeros-one decision variables. The coefficients $\bar{a}$, $\bar{b}$, and $\bar{c}$ are expressed as fuzzy numbers. i and j indices refer to bus index and the constraint index. n and m are the numbers of system buses and considered constraints.

The solution method of the FBLP presented in (11) and (12), maximizes λ, where:

$$\lambda = \min\{\mu_1, \mu_2, \ldots\ldots\ldots\ldots\ldots, \mu_i\} \tag{13}$$

where μ_i are the membership functions of all the functions (control and dependent variables and objective), within the zeros-ones range. Hence, the fuzzy based PMU allocation problem is turned to maximizing λ, as follows:

$$\text{Maximize } \lambda, \text{ s.t.} \tag{14}$$

$$\lambda \le \mu_i(\cdot), i = 1, 2, 3 \ldots\ldots n_c \tag{15}$$

where, n_c is the numbers of objective and constraints of the PMU allocation problem.

Also, with the aid of the fuzzy modeling presented earlier, Eqs. (14) and (15) can be turned into maximizing λ, subjected to

$$\sum_{i=1}^{n_b} \omega_i x_i + (n_b - 1)\lambda \le n_b \tag{16}$$

$$x_i + \lambda \le 1 \tag{17}$$

$$\left(y_i^{\max} - 1\right)\lambda - y_i \le -1 \quad 1 \le y_i \le y_i^{\max} \tag{18}$$

$$\left(y_i^{\max} - 2\right)\lambda - y_i \le -2 \quad 2 \le y_i \le y_i^{\max} \tag{19}$$

Furthermore, four evaluation indices are suggested to measure the effectiveness of the proposed PMU allocation procedure. The

Table 1
Test system data.

Test system	Number of ZIBs	ZIB locations	Number of lines/branches
IEEE 14-bus	1	7	20
IEEE 30-bus	6	6,9,22,25,27,28	41
England 39-bus	12	1,2,5,6,9,10,11,13,14,17,19,22	46
IEEE 57-bus	15	4,7,11,21,22,24,26,34,36,37,39,40,45,46,48	78
IEEE 118-bus	10	5,9,30,37,38,63,64,68,71,81	179

Table 2
Optimal number of PMUs and channels under normal operating conditions.

System	Number of PMUs	Optimal PMU locations	Total PMU channels
IEEE 14-bus	3	2,6,9	15
IEEE 30-bus	7	1,2,10,12,15,19,27	34
England 39-bus	8	3,8,12,16,20,23,25,29	28
IEEE 57-bus	11	1,4,13,20,25,29,32,38,51,54,56	48
IEEE 118-bus	28	2,8,11,12,17,21,25,28,33,34,40,45,49,52, 56,62,72,75,77,80,85,86,90,94,101,105,110,114	115

first performance index PI_1 measures the PMUs penetration or percentage of buses with allocated PMUs (n_{PMU}) that reaches complete observability of the studied system relative to the system size. This index is affected by the system configuration and the system size.

$$PI_1 = \frac{n_{PMU}}{n_b} \tag{20}$$

The second performance index PI_2 measures the sufficient total PMU channels ($n_{PMU}^{Channel}$) with respect to the system size.

$$PI_2 = \frac{n_{PMU}^{Channel}}{n_b} \tag{21}$$

The third evaluation index PI_3 measures the percentage of the required voltage channels ($n_{Channel}^{v}$) to the system size. Finally, the fourth index PI_4 gives the percentage of the channels $n_{Channel}^{i}$ which are specified for current measurements to number of lines of the system (n_l). The suggested evaluation indices considerably reflect the goodness of the solution of the optimization problem.

$$PI_3 = \frac{n_{Channel}^{v}}{n_b} \tag{22}$$

$$PI_4 = \frac{n_{Channel}^{i}}{n_l} \tag{23}$$

Results and discussion

Test systems

The proposed methodology using the fuzzy-based model is applied to five IEEE benchmark systems to solve the optimal PMU placement problem. The considered systems are the IEEE 14-bus, 30-bus, 39-bus of the New England network, 57-bus and 118-bus test system [52,53]. Table 1 shows the number and locations of zero injection buses (ZIBs) and the branch number for the test systems [29]. The results using the proposed algorithm are compared with corresponding results of other optimization methods found in the literature.

Results under normal operating conditions

Table 2 shows the optimal number of PMUs, their location, and the optimal number of measurement channels for each PMU, which lead to the full observability of all test systems under normal operating conditions with a reduced number of total measurement channels. It can be noted that the number of PMUs increases with the number of system buses and the total number of channels; otherwise, the number of PMUs diminishes with increasing ZIBs in the power system.

The minimum numbers of allocated PMUs in terms of the total bus number, for the tested systems, are 21.42% for the IEEE 14-bus, 23.33% for the IEEE 30-bus test system, 20.51% for the 39-bus test system, 19.29% for IEEE 57-bus test system, and 23.73% for the IEEE 118-bus test system. Hence, one can conclude that the optimal PMUs penetration under normal operating conditions is around 20% of the system size.

Results under system contingencies

Table 3 shows the optimal number of PMUs with the minimum number of measurement channels that give each test system full observability under system contingencies (single line/PMU outage). Reasonably, the number of PMUs increases compared to the corresponding results under normal operating conditions. It is noticed that the allocated PMUs penetration, in terms of the total bus number of the tested systems, varies within the range of 38% for the IEEE 57-bus to 50% for the rest. Hence, according to the study, the optimal PMU allocated under abnormal conditions is almost double its value under normal conditions.

Figs. 5 and 6 demonstrate the allocation of PMUs and number of channels under different operating conditions for both normal and abnormal operating conditions, respectively.

Evaluation of the proposed solution

Assessment using the suggested performance indices: Table 4 presents the proposed evaluation indices for both normal and abnormal conditions. The first performance index varies between 19.29% and 23.72% for the tested power systems in normal operating conditions, while the range of this index increases at the abnormal conditions for all tested systems to be within 38.59% to 50.84%. This index means at least 20% and 40% of system buses should be covered with PMU devices to achieve the acceptable observability criterion at normal and abnormal operating conditions, respectively. As the PI_1 in a real power grid is around 30%, this means the power grid is not fully observed under abnormal operating conditions.

The second performance index, which measures the ratio between the numbers of PMU channels to the system size, is varied from 71.8% to 113%, and from 130.77% to 221.4% for the tested power systems at normal and abnormal conditions, respectively. As obvious, the second performance index is almost doubled at abnormalities which reflect the need to increase the channels number

Table 3
Optimal number of PMUs and channels under abnormal operating conditions.

System	Number of PMUs	Optimal PMU locations	Total PMU channels
IEEE 14-bus	7	1,2,4,6,9,10,13	31
IEEE 30-bus	14	1,2,3,7,10,12,13,15,17,18,19,24,27,30	55
England 39-bus	18	4,8,16,18,20,23,25,26,29,30,31,32,33,34,35,36,37,38	51
IEEE 57-bus	22	1,2,4,9,12,15,18,19,25,28,29,30,32,33,38,41, 47, 50,51,53,54,56	88
IEEE 118-bus	60	2,3,6,8,10,11,12,15,17,19,21,22,24,25,27,28,29, 32,34,36,40,42,43, 45,46,49,51,52,54,56,57,59,62, 66,70,72,75,76,77,79,80,84,85,86,87,89,90,92,94, 96,100,101,105,107,108,110,111,112,115,117	229

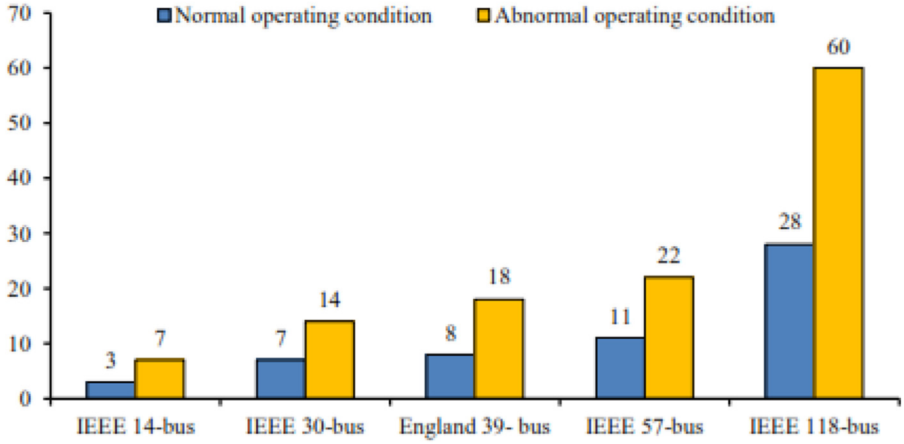

Fig. 5. PMUs allocation at different operating conditions using fuzzy based procedure.

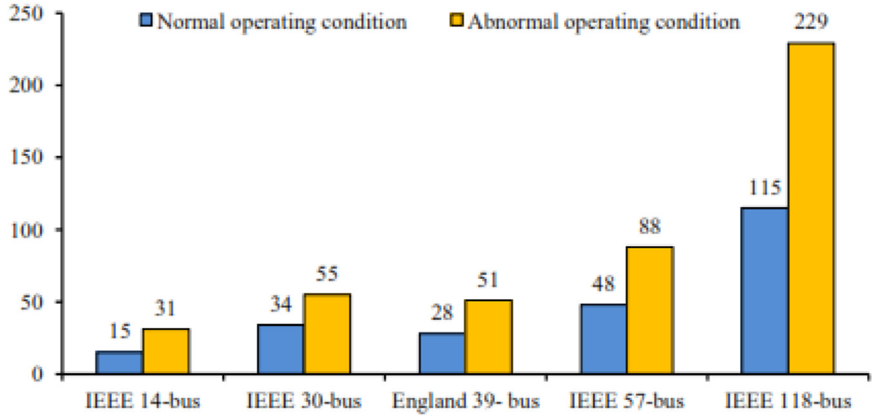

Fig. 6. PMU channels at different operating conditions using fuzzy based procedure.

Table 4
Performance indices at different operating conditions.

System	Normal operating conditions				Abnormal operating condition			
	PI_1	PI_2	PI_3	PI_4	PI_1	PI_2	PI_3	PI_4
IEEE 14-bus	21.43%	1.0714	0.2143	0.6	50%	2.214	0.5	1.2
IEEE 30-bus	23.33%	1.1333	0.2333	0.6585	46.67%	1.833	0.4667	1
England 39-bus	20.51%	0.718	0.2051	0.5263	46.15%	1.3077	0.4615	0.868
IEEE 57-bus	19.29%	0.8421	0.1929	0.4231	38.59%	1.5439	0.3859	0.8461
IEEE 118-bus	23.72%	0.9745	0.2372	0.486	50.84%	1.915	0.5084	0.9441

during the abnormal operating conditions in a view to preserving acceptable measurement redundancy.

The third index measures the percentage of the number of channels which are assigned for voltage measuring. It is concluded that variation of order 2 is required to effectively measure the voltage signals. Numerically, both the first and the third performance indices are typical. The last performance index measures the relation between the current channels with the transmission line number. The smallest and largest values of the fourth index equal 0.4231 and 0.6585 at the normal operating condition, and equal 0.8461 and 1.2 at the abnormal condition, respectively.

Table 5
Comparison results for various test systems using different algorithms at normal operating conditions.

Algorithm	Test systems				
	14-bus	30-bus	39-bus	57-bus	118-bus
Proposed fuzzy based procedure	3	7	8	11	28
Random component outage [3]	4	–	13	–	32
Binary PSO (BPSO) [4]	4	9	9	11	32
Integer linear programming by CEPLEX [13]	3	7	8	11	28
Depth First search [18]	6	–	16	–	41
Simulated Annealing Method [18]	4	–	9	–	29
Minimum spanning tree [18]	3	–	9	–	31
Non dominated sorting GA [29]	3	7	–	12	29
Multistage procedure [29]	3	7	8	11	28
Graph theoretic procedure [29]	5	11	–	19	38
Tabu search [29]	3	–	10	13	–
Topology based GA [29]	3	7	8	11	29
Branch and Bound [29]	3	7	9	12	29
Memetic Algorithms [39]	3	7		12	29
Binary integer linear programming (BILP) [43]	4	7	–	13	29
Modified binary PSO [44]	3	7	8	12	29
Binary Imperialistic competition algorithm [45]	3	7	–	11	28
Hybrid two-phase search algorithm [47]	3	8	–	11	28
Integer programming based procedure [48]	3	7		12	28
Binary search algorithm [49]	3	7	8	–	–
Modified GA [52]	3	7	8	11	29

Table 6
Optimal PMUs locations for various test systems at system contingencies.

Algorithm	Test systems				
	14-bus	30-bus	39-bus	57-bus	118-bus
Proposed fuzzy based procedure	7	14	18	22	60
Binary integer programming [12]	–	15		25	61
CPLEX [13]	8	17	22	26	65
Single shot N-1 security [18]	8	–	18	–	72
Multistage procedure [19]	7	15	–	26	64
BILP [43]	9	16	–	25	61
BPSO [44]	7	15	17	22	62
BICA [45]	7	13	–	22	61
ILP [46]	–	–	–	29	64
Integer programming based procedure [48]	7	17	–	26	65
Binary search algorithm [49]	7	14	–	–	–
Modified GA [52]	7	14	17	25	61

Comparative evaluation: Table 5 presents a comparison between the optimal number of PMUs that is obtained using the proposed framework and other studies reported in the literature under normal operating conditions. This comparison reflects the significant capability of the proposed method in finding the optimal number of PMUs with the lowest number of measurement channels which will reflect on the required installation cost to achieve full system observability.

Table 6 presents a comparison between the optimal number of PMUs that is obtained using the proposed method and those of the other previously published methods under system contingencies. As shown, the optimal number of PMUs that obtained by using the proposed framework is similar to the best corresponding results presented in the other previous works, taking into account the additional limitations of measurement channel number that are considered in this study.

Finally, Tables 5 and 6 validate the ability of the proposed framework in finding the optimal number of PMUs at minimum channels number. Therefore, minimum installation costs are achieved to make small and large-scale power systems fully observable under contingencies.

Conclusion

In this paper, a novel fuzzy-based procedure to find the optimal PMU locations while considering the minimum number of PMU measuring channels for completing topological network observability, is proposed. The proposed method is practiced under normal operation conditions as well as system contingencies. Besides, several reduction rules were developed to reduce the number of measuring channels while maintaining complete system observability. The following main contributions are highlighted:

- √ An investigated linear fuzzy modeling of the PMU placement problem is carried out at different operating conditions.
- √ The fuzzy constraints are ramped according to the operating requirements and measurement redundancy.
- √ The novelty of the proposed procedure has proven the capability of the proposed linear fuzzy models to find better (or the same) optimal number of PMUs with lower number of channels compared to previous algorithms in the literature under various operating conditions.
- √ The proposed solution method was validated on five IEEE benchmark systems for different operating conditions.
- √ The full system observability is satisfied in normal and emergency operating modes.
- √ The PMU channels are integrated to the conventional PMU placement method.
- √ Four performance indices are suggested to measure the solution quality. These indices assure the observability and the measurement criteria.
- √ It was concluded from the presented results that the required PMUs penetrations to satisfy full system observability are

20% and 50% of the system size under normal and abnormal operating conditions, respectively.

Finally, the proposed method may represent a potential tool to monitor power systems and expand the facilities that allow enhancement of the system performance [54]. Also, this will help the operators in a smart grid environment as the proposed fuzzy-based allocation procedure is characterized by its novelty and simplicity to be modeled and solved with multiple objective and constraints.

Conflict of interest

The authors declare no conflict of interest.

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