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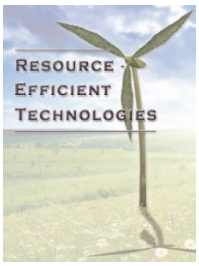

Research Paper

QUANTUM AND CLASSICAL EFFECTS AT SCATTERING OF HIGH ENERGY CHARGED PARTICLES IN THIN CRYSTALS

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Abstract

The present work reviews the results concerning quantum scattering theory of ultrarelativistic electrons in ultrathin crystals and its comparison with analogous classical results. It deals with an intermediate range of thicknesses, large enough for that the particle motion could not be considered as rectilinear but small enough for that the channeling regime of motion was not established. The quantum theory is based both upon the representation of the scattering amplitude as an integral over the surface surrounding the target, and on the so-called operator method of determination of the wave function as a solution of a Schrödinger-like equation. The latter method implies a wide use of the Fourier technique, both in calculation of each next step in the wave packet evolution, and in moving from the spatial coordinates to the angular ones. The authors compare the quantum differential scattering cross-sections with the classical ones in the considered range of crystal thicknesses and show their resemblances, distinctions and the evolution of these distinctions with the change of the particle energy. The simplest variant of quantum scattering theory based upon the eikonal approximation of quantum mechanics is considered. In the paper the quantum differential scattering cross-section was calculated and its affinity with the classical one was demonstrated. In the preparation of these lecture notes the material of the paper [4] was used.

Key words: relativistic electrons, quantum scattering theory, angular distributions, eikonal approximation, thin crystal.

1. Introduction

When fast charged particles pass through crystals the phenomenon of channeling is possible, at which the particles move inside the channels formed by crystal atomic strings or atomic planes, being periodically deviated to small angles from the channel direction [1, 2]. In ultrathin crystals there is no room for channeling phenomenon to develop (see Fig. 1). However, there remains the possibility of appearance of several coherence and interference effects at the interaction of particles with crystal atoms (see [2, 3] and references therein).

The work presents some results of classical and quantum theories of high energy charged particles scattering in transitional range of crystal thicknesses, from those thicknesses at which the channeling phenomenon is not developed up to those at which this regime of motion is established [4–6]. Quantum theory is based upon special representation of the scattering amplitude [7] in the form of an integral over

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the surface surrounding the region of influence of the external crystal field onto the particle (in the considered problem this corresponds to the field of the entire crystal), and upon the development of numeric methods of calculating of the wave function inside the crystal, that is done by using the so-called spectral method of solving wave equations [8–10]. The classical theory is based upon the solution of the particle motion equation by numerical methods [11]. The main attention is paid to the comparative analysis of quantum and classical characteristics of the scattering process at different crystal thicknesses and particle energies.

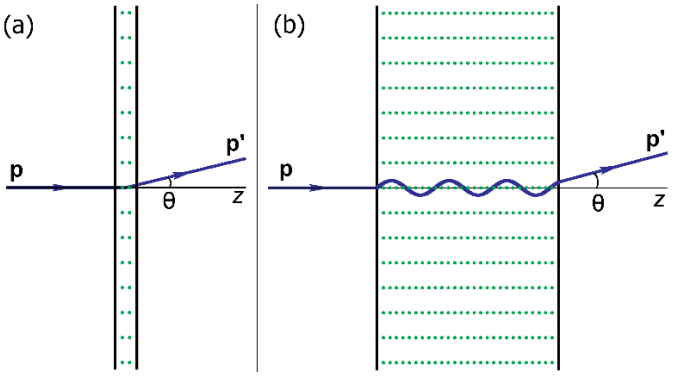

Fig. 1. *Scattering in absence of channeling (a) and in the channeling regime (b)*

2. Quantum scattering theory of ultrarelativistic electrons in external field

Let us consider the scattering of relativistic electrons incident onto a thin crystal along one of its crystal axes. The differential scattering cross-section and the scattering amplitude in this case are defined by the following formulas [2]:

$$\frac{d\sigma}{do} = \left|a(\vartheta)\right|^2, \tag{A.1}$$

$$a(\vartheta) = -\frac{1}{4\pi\hbar^2}\int_V d^3r\, e^{-\frac{i}{\hbar}\mathbf{p}'\mathbf{r}}\,\overline{u'}\gamma_0 U(\mathbf{r})\psi(\mathbf{r}), \tag{A.2}$$

where ϑ is the scattering angle; $\psi(\mathbf{r})$ – the wave function of the electron passing through the crystal, $\overline{u}'$and $\mathbf{p}'$ – bispinor and momentum of the scattered electron respectively, and $U(\mathbf{r})$ – the potential energy of the interaction of the electron with the crystal lattice field (the system of units in which the light velocity is equal to one, c=1, is used). The integration in (A.2) is performed inside the volume V where the particle is subject to the external field action.

At the incidence of fast particles onto the crystal along one of its axes (named z axis) the correlations between consequent collisions of the particle with lattice atoms are substantial. As a result of these correlations, the particle motion is mainly determined by the continuous potential of crystal atomic strings situated parallel to the z axis, so the lattice potential averaged along this axis is [1, 2]:

$$U_c(\boldsymbol{\rho}) = \frac{1}{L}\int_0^L dz\, U(\mathbf{r}), \qquad 0 \le z \le L, \tag{A.3}$$

where L is the crystal thickness and $\boldsymbol{\rho}$ – the coordinates (x,y) in the plane orthogonal to the z axis (outside the crystal $U_c(\boldsymbol{\rho})$=0).

By using the Dirac equation for the wave function of the electron in the field $U(\mathbf{r})$:

$$(\varepsilon\gamma_0 + i\hbar\boldsymbol{\gamma}\nabla - m)\psi = \gamma_0 U(\mathbf{r})\,\psi \tag{A.4}$$

the scattering amplitude (A.2) is as follows

$$a(\vartheta) = -\frac{i}{4\pi\hbar}\int_V d^3r\,\mathrm{div}\left(\overline{u'}\boldsymbol{\gamma}\,\psi(\mathbf{r})e^{-\frac{i}{\hbar}\mathbf{p}'\mathbf{r}}\right). \tag{A.5}$$

The scattering amplitude in this case, with the use of the Gauss theorem, can be presented in the form of the integral over a closed surface surrounding the external field region [7]:

$$a(\vartheta) = -\frac{i}{4\pi\hbar}\oint d\mathbf{S}\,\overline{u'}\boldsymbol{\gamma}\,\psi(\mathbf{r})e^{-\frac{i}{\hbar}\mathbf{p}'\mathbf{r}}, \tag{A.6}$$

where $d\mathbf{S}$ is an element of the surface surrounding the crystal.

It is essential that the surface integral in (A.6) does not depend on the surface form, so as the only requirement imposed on this surface is that it surrounds the entire area of the external field action. In the considered problem it is convenient to choose as such a closed surface a cylinder whose bases coincide with crystal sides. By neglecting the contribution of the cylinder lateral side parallel to the z axis in the surface integral (A.6), the expression for the scattering amplitude is as follows:

$$a(\vartheta) = -\frac{i}{4\pi\hbar}\int d^2\rho\, e^{-\frac{i}{\hbar}\mathbf{p}'\mathbf{r}}\,\overline{u'}\gamma_z\,\psi(\mathbf{r})\Big|_{z=0}^{z=L}. \tag{A.7}$$

For determining the wave function of the electron in the field $U_c(\boldsymbol{\rho})$ the squared Dirac equation [2] is used:

$$\left[\begin{matrix}(\varepsilon - U_c(\boldsymbol{\rho}))^2 - (i\hbar\nabla)^2 - \\ -m^2 + i\hbar\boldsymbol{\alpha}\nabla U_c(\boldsymbol{\rho})\end{matrix}\right]\psi(\mathbf{r}) = 0, \tag{A.8}$$

where m and ε are particle mass and energy respectively, and $\boldsymbol{\alpha}=\gamma_0\boldsymbol{\gamma}$.

Before entering into the crystal, the electron wave function is represented as a plane wave characterized by momentum $\mathbf{p}$ and bispinor u_p. Then, by detaching from $\psi(\mathbf{r})$ the bispinor u_p and the plane wave factor (defined $\psi(\mathbf{r}) \equiv \varphi(\mathbf{r})u_p\, e^{\frac{i}{\hbar}\mathbf{p}\mathbf{r}}$), the equation for $\varphi(\mathbf{r})$ is as follows:

$$i\hbar v\partial_z\varphi = \left[\frac{\hat{\mathbf{p}}^2}{2\varepsilon} + U_c(\boldsymbol{\rho}) - \frac{i\hbar}{2\varepsilon}\boldsymbol{\alpha}\nabla U_c(\boldsymbol{\rho}) - \frac{U_c^2(\boldsymbol{\rho})}{2\varepsilon}\right]\varphi(\mathbf{r}), \quad (A.9)$$

where $v=p/\varepsilon$ is the velocity of the electron and $\hat{\mathbf{p}}=-i\hbar\nabla$ – operator of momentum.

3. Eikonal approximation in quantum scattering theory

On the basis of the above formulas it is possible to study the scattering of fast charged particles in the fields with a complicated configuration, such as the field in the crystal. In this case the development of approximated methods is crucial as that enables to calculate the wave function avoiding the use of specific form of the potential of the particle interaction with the external field, just as the development of numeric methods of calculation of the wave function.

One of the simplest methods to perform the calculations of the wave function and the scattering amplitude at small angles of fast charged particles in the external field is the one based upon the eikonal approximation of quantum electrodynamics. Let us give some results that can be obtained for the scattering cross-section based upon this method.

In the eikonal approximation of quantum electrodynamics the solution of the equation (A.9) is as follows [2]:

$$\varphi(\boldsymbol{c},z) = e^{\frac{i}{\hbar}\chi(\boldsymbol{c},z)}, \quad (A.10)$$

where $\chi(\boldsymbol{c},z) = -\frac{1}{v}\int_{-\infty}^{z} dz' U(\boldsymbol{c},z')$. By substituting this solution into (A.5), it is found that

$$a(\mathbf{q}) = -i\frac{\overline{u'}\gamma_z u_p}{4\pi\hbar}\int d^2\rho\, e^{\frac{i}{\hbar}\mathbf{q}_\perp\boldsymbol{c}}\left(e^{\frac{i}{\hbar}\chi(\boldsymbol{c})} - 1\right), \quad (A.11)$$

where u' is the scattered electron's bispinor, $\mathbf{q}=\mathbf{p}-\mathbf{p}'$ is the momentum transmitted to the external field, and

$$\chi(\boldsymbol{c}) = -\frac{1}{v}\int_{-\infty}^{\infty} dz\, U(\boldsymbol{c},z). \quad (A.12)$$

It should be noted that in deduction of the formula (A.10) the terms proportional to ε^{-1} were dropped [2]. With the same precision as in (A.11), the term $q_zL\approx q_\perp^2/\varepsilon$ in the exponent index was also dropped. So, the formula (A.11) is true for the scattering of particles at small angles. Moreover, for such angles $\overline{u'}\gamma_z u_p\approx 2p$ and then, in this range of scattering angles spin effects in scattering can also be neglected. Hence, the scattering amplitude takes the following form:

$$a(\mathbf{q}_\perp) = -i\frac{p}{2\pi\hbar}\int d^2\rho\, e^{i\mathbf{q}_\perp\boldsymbol{c}/\hbar}\left(e^{\frac{i}{\hbar}\chi(\boldsymbol{c})} - 1\right). \quad (A.13)$$

If the condition $|\chi(\boldsymbol{\rho})|<<\hbar$ is fulfilled, the exponential factor in (A.13) can be expanded in series about $\chi/\hbar$. In the first approximation to this parameter, the scattering amplitude transforms into the corresponding result of the first Born approximation,

$$a_B(\mathbf{q}_\perp) \approx \frac{p}{2\pi\hbar^2}\chi_{\mathbf{q}_\perp}, \quad (A.14)$$

where $\xi_{\mathbf{q}_\perp}$ is the Fourier component of ξ_ρ.

At arbitrary values of the parameter $\chi/\hbar$ the scattering cross-section to a non-zero angle can be presented, according to (A.1) and (A.13), as follows:

$$\frac{d^2\sigma}{d^2\mathbf{q}_\perp} = \frac{1}{4\pi^2\hbar^2}\int d^2\rho\, d^2\rho' e^{\frac{i}{\hbar}\mathbf{q}_\perp(\boldsymbol{c}-\boldsymbol{c}')} e^{\frac{i}{\hbar}[\chi(\boldsymbol{c})-\chi(\boldsymbol{c}')]}. \quad (A.15)$$

In the quasiclassical approximation, when $|\chi(\rho)|>>\hbar$, the calculation of integrals entering into (A.13) can be carried out on the basis of the stationary phase method (see, e. g., [2]). In this case, if the main contribution in the scattering cross-section (A.15) is made by the values of $\boldsymbol{\rho}$ lying in the vicinity of $\boldsymbol{\rho}'$, the formula (A.15) transforms into the corresponding result of the classical scattering theory. In fact, in this case the expansion about $\boldsymbol{\rho}-\boldsymbol{\rho}'$ can be performed in the phase part of the exponent, $[\chi(\boldsymbol{\rho})-\chi(\boldsymbol{\rho}')]$. In the first approximation of such expansion it is found that

$$\frac{d^2\sigma}{d^2\mathbf{q}_\perp} \approx \approx \frac{i}{4\pi^2\hbar^2}\int d^2\rho'\int d^2\Delta \exp\left[\frac{i}{\hbar}\left(\mathbf{q}_\perp + \frac{\partial\chi(\boldsymbol{c}')}{\partial\boldsymbol{c}'}\right)\Delta\right], \quad (A.16)$$

where $\Delta=\boldsymbol{\rho}-\boldsymbol{\rho}'$ and, consequently,

$$\frac{d^2\sigma}{d^2q_\perp} = \int d^2\rho'\,\delta(\mathbf{q}_\perp - \mathbf{q}_\perp(\boldsymbol{c}')), \quad (A.17)$$

where $\mathbf{q}_\perp(\boldsymbol{c}') = -\frac{\partial}{\partial\boldsymbol{c}'}\chi(\boldsymbol{c}')$. In passing in (A.17) from the integration over $\mathbf{q}_\perp(\boldsymbol{\rho}')$ to the integration over $\boldsymbol{\rho}'$, the classical scattering cross-section is obtained

$$\frac{d^2\sigma}{d^2q_\perp} = \frac{1}{\left|\partial\mathbf{q}_\perp(\boldsymbol{c}')/\partial\boldsymbol{c}'\right|_{\mathbf{q}_\perp(\boldsymbol{c}')=\mathbf{q}_\perp}}, \quad (A.18)$$

where $|\partial\mathbf{q}_\perp(\boldsymbol{\rho}')/\partial\boldsymbol{\rho}'|$ is the transition determinant from $\mathbf{q}_\perp(\boldsymbol{\rho}')$ to $\boldsymbol{\rho}'$. The value $\mathbf{q}_\perp(\boldsymbol{\rho}')=\mathbf{q}_\perp$ is the deflection function of fast charged particle in non-symmetric field for scattering at small angles.

4. Operator method in the scattering theory

Characteristic values for the scattering angles of high energy electrons in the thin crystal are small. In this case, by resolving the Eq. (A.9), spin effects in

scattering (spin-field interaction) can be neglected, so far as the terms proportional to $U^2/2\varepsilon$ and $\hat{p}_z^2/2\varepsilon$.

As a result, the equation for $\varphi(\mathbf{r})$ is as follows:

$$i\hbar v\partial_z\varphi = \left[\frac{\hat{\mathbf{p}}_\perp^2}{2\varepsilon} + U_c(\boldsymbol{c})\right]\varphi, \qquad (A.19)$$

where $\hat{\mathbf{p}}_\perp = -i\hbar\dfrac{\partial}{\partial \boldsymbol{c}}$.

The equation (A.19) is a Schrödinger-like equation, where the particle mass is replaced by the energy and z/v is used instead of time, so it can be solved with the help of numerical method developed in [8]. Relying on the analogy of equation (A.19) with the Schrödinger equation, the evolution of the wave function with depth δz of particle's penetration into the crystal is expressed, according to (A.19), in the following operator form:

$$\varphi(\boldsymbol{c}, z+\delta z) = \exp\left\{-i\frac{\delta z}{\hbar v}\left[\frac{\hat{\mathbf{p}}_\perp^2}{2\varepsilon} + U_c(\boldsymbol{c})\right]\right\}\varphi(\boldsymbol{c}, z). \qquad (A.20)$$

This expression is formally exact for any δz, but its direct application leads to mathematical difficulties connected with the fact that the exponent index in (A.20) is composed of two non-commuting terms.

However, applying the operator technique [12] at small values of δz makes it possible to rewrite (A.20) in approximated form in which the commutators of the order higher than $[\hat{A},\hat{B}]$ are neglected, where

$$\hat{A} = -(i/\hbar v)(\hat{\mathbf{p}}_\perp^2/2\varepsilon) \text{ and } \hat{B} = -(i/\hbar v)U_c(\boldsymbol{c}).$$

Hence, with the precision up to the terms proportional to δz^3, such Zassenhaus product formula is true:

$$\exp(\delta z(\hat{A}+\hat{B})) \approx \exp\left(\delta z\frac{\hat{B}}{2}\right)\exp(\delta z\hat{A})\exp\left(\delta z\frac{\hat{B}}{2}\right). \qquad (A.21)$$

By dividing the full crystal thickness L into a large number of small intervals δz, so as for each of them (A.21) is accomplished, (A.19) for a given L is solved by the way of iteration procedure.

By using the solution for $\varphi(\boldsymbol{\rho},z)$ obtained in such a manner at $z=L$, the scattering cross-section and the scattering amplitude can be written in the form:

$$\frac{d\sigma}{d\vartheta^2} = \left|a_L(\mathbf{q}_\perp)\right|^2, \qquad (A.22)$$

$$a_L(\mathbf{q}_\perp) = -\frac{ip}{2\pi\hbar}\int d^2\rho\, e^{\frac{i}{\hbar}\mathbf{q}_\perp\mathbf{c}}\varphi(\boldsymbol{c}, L), \qquad (A.23)$$

where $\mathbf{q}_\perp = p\boldsymbol{\vartheta}$ is the transversal component of the momentum transmitted to the crystal and $\boldsymbol{\vartheta}=(\vartheta_x,\vartheta_y)$ is the scattering angle ($\vartheta<<1$). Here the fact that at small scattering angles $\bar{u}'\gamma_z u_p \approx 2p_z \approx 2p$ was used.

It is worth mentioning that if in the exponent (A.21) the terms containing the operator $\hat{A}$ are neglected, the wave function (A.20) and the scattering amplitude (A.23) in the eikonal approximation of quantum scattering theory will be obtained. At sufficiently large energies of the electron ($\varepsilon\to\infty$) this condition can always be accomplished. Taking this into account leads to the need for numerical methods to calculate both the wave function and the scattering cross-section. At this procedure, the calculation of action of the differential operator $\hat{\mathbf{p}}_\perp^2$ in the exponent (A.20) is performed by means of Fourier technique.

5. Classical theory

Now let us consider the fast electrons scattering in the thin crystal on the basis of classical mechanics. The particle motion in this case is determined by its classical trajectory that is the solution of classical equations of motion and satisfies to the given initial conditions.

The differential scattering cross-section in classical mechanics corresponds to the elementary surface in the impact parameter space, from which the particle is scattered into the elementary solid angle $do \approx d2\vartheta$:

$$d\sigma_{cl} = d^2b(\boldsymbol{\vartheta}). \qquad (A.24)$$

For calculating this cross-section it is necessary to find the deflection function of the particle in the external field $\boldsymbol{\vartheta}=\boldsymbol{\vartheta}(\mathbf{b})$, that is the dependence of the particle scattering angle $\boldsymbol{\vartheta}=(\vartheta_x,\vartheta_y)$ on the impact parameter $\mathbf{b}=(b_x,b_y)$, and then to perform the inversion of this function, i.e. to define the dependence $\mathbf{b}=\mathbf{b}(\boldsymbol{\vartheta})$ (see, e. g. [2, 13]). In the complex field, e. g. inside the crystal, the deflection function $\boldsymbol{\vartheta}=\boldsymbol{\vartheta}(\mathbf{b})$ is quite a complex function of the coordinates b_x and b_y. It is important that the deflection function inversion is not single-valued in the common case. Taking into account this ambiguity, the classical scattering cross-section (A.24) can be presented in the following form:

$$d\sigma(\boldsymbol{\vartheta}) = \sum_n d^2b_n(\boldsymbol{\vartheta}) = \sum_n \left.\frac{1}{\left|\partial\boldsymbol{\vartheta}/\partial\mathbf{b}\right|_n}\right|_{\mathbf{b}=\mathbf{b}_n(\boldsymbol{\vartheta})} d^2\vartheta, \qquad (A.25)$$

where $|\partial\boldsymbol{\vartheta}/\partial\mathbf{b}| = |\partial(\vartheta_x,\vartheta_y)/\partial(b_x,b_y)|$ is the determinant of transition from the variables $\vartheta_x=\vartheta_x(b_x,b_y)$ and $\vartheta_y=\vartheta_y(b_x,b_y)$ to b_x and b_y, with the subsequent inversion of the deflection function. The summation in (A.25) runs over n single-valued branches of the deflection function.

The formula (A.25) for the scattering cross-section can also be written in the form:

$$\frac{d\sigma_{cl}(\boldsymbol{\vartheta})}{d^2\vartheta} = \int d^2b\ \delta(\boldsymbol{\vartheta} - \boldsymbol{\vartheta}(\mathbf{b})). \qquad (A.26)$$

It should be pointed out that this expression for the classical scattering cross-section can be easily

obtained from the quantum one (A.22) in the frames of the quasi-classical approximation of quantum mechanics, in the case of a single-valued correspondence between the particle's scattering angle and its impact parameter.

In the common case the deflection function can be expressed via classical particle trajectories after their exit from the external field region (in the given case from the crystal). At small scattering angles this dependence is defined by the relation

$$\boldsymbol{\vartheta}(\mathbf{b}, L) = \frac{\mathbf{v}_\perp(\mathbf{b}, L)}{v}, \tag{A.27}$$

where $\mathbf{v}_\perp(\mathbf{b},L)$ is the transversal component of the particle velocity at $z=L$. The velocity $\mathbf{v}_\perp(\mathbf{b},z)$ is defined as the solution of the classical equation of motion. At the particle motion in the continuous strings potential (A.3), this equation, with the precision up to the terms of the order of $\mathcal{O}(v_\perp^2/v^2)$, has the following form [1, 2]:

$$\ddot{\boldsymbol{c}} = -\frac{1}{\varepsilon}\frac{\partial}{\partial \boldsymbol{c}} U_c(\boldsymbol{c}). \tag{A.28}$$

Its solution can be found on the basis of numerical methods (see, e. g., [11]) for a large number of particles incident onto the crystal at different values of impact parameters. This enables to develop the procedure of numerical calculation of the scattering cross-section for a particle beam falling onto the crystal with random uniformly distributed impact parameters. In this case, the probability of the particles scattering into the solid angle interval $(\boldsymbol{\vartheta},\boldsymbol{\vartheta}+d\boldsymbol{\vartheta})$ is determined by the relation of particle number $dN(\boldsymbol{\vartheta})$ that exited into this interval, to the full number of incident particles N. The scattering cross-section is connected with dN by the relation

$$dN(\boldsymbol{\vartheta}) = \frac{N}{S} d\sigma_{cl}(\boldsymbol{\vartheta}), \tag{A.29}$$

where S is the transverse size of the crystal surface which interacts with the beam.

6. Comparative analysis of quantum and classical effects in scattering

On the basis of the above-stated methods, it is possible to carry out numerical calculations of quantum and classical elastic scattering cross-sections of relativistic charged particles in thin crystals, and perform comparative analysis of quantum and classical effects in scattering. The paper presents some results of calculations for relativistic electrons with different energies in ultrathin Si crystals that are oriented by their ⟨100⟩ axis to the incident beam. The continuous potential of the whole ensemble of atomic strings represents quite a complicated two-dimensional periodical function of the coordinates (x,y) in the plane orthogonal to the z axis (see Fig. 6.16 in [2]). The calculation of quantum and classical scattering cross-sections in such a field can only be performed on the basis of numerical methods.

In quantum calculations, a wide wave packet covering a large number of crystal atomic strings whose axes are periodically situated in the transverse plane was used as the initial state $\varphi(\boldsymbol{\rho},z=0)$.

The calculations based upon the classical theory were performed for 10^7 particles incident onto the crystal with the impact parameters randomly distributed over the elementary cell in its transverse plane.

Fig. 2 shows the results of calculations of quantum and classical scattering cross-sections for ultrarelativistic electrons in Si crystals in the transitional thickness range, at which the channeling phenomenon is not developed or is in the process of development.

Unlike classical scattering cross-section, the quantum one, as proved by calculations, contains sharp peaks at some values of scattering angles. These peaks are caused by the interference effect at the wide wave packet (plane wave) scattering on crystal atomic strings that are periodically situated inside the crystal. Due to this interference, the transmitted momentum values at particle scattering on the crystal take the values proportional to the whole number of inverse lattice vectors **g**. Here it is essential that the positions of peaks in angular distributions of particles depend on the character of distribution of atomic strings in the transverse plane. In this regard, it is assumed that the analysis of angular distributions of particles scattered on the thin crystal is analogical to the X-Ray analysis of the crystal structure, but with such a difference that in the given case the authors deal not with single crystal atoms but with atomic strings as elementary scattering objects.

As the full particle energy increases, the distance between interference maxima decreases, and the quantum scattering picture approaches to the classical one.

It should be noted that the quantum formulas for the scattering cross-section (A.1), (A.2) are true for macroscopic distances between the target and the detectors ($r\to\infty$). The value L entering in the formula (A.7) determines, in its turn, not the distance between the target and the detector but the size of the area in which the interaction of particles with the external field takes place. There is, however, a number of factors that can influence the thin structure of angular distributions of scattered particles in the experiment, such as the angular divergence of the incident beam and the finite linear width of the beam. In particular, for the results obtained in the work to be detectable, it is required that the detection of scattered particles was performed in the distance

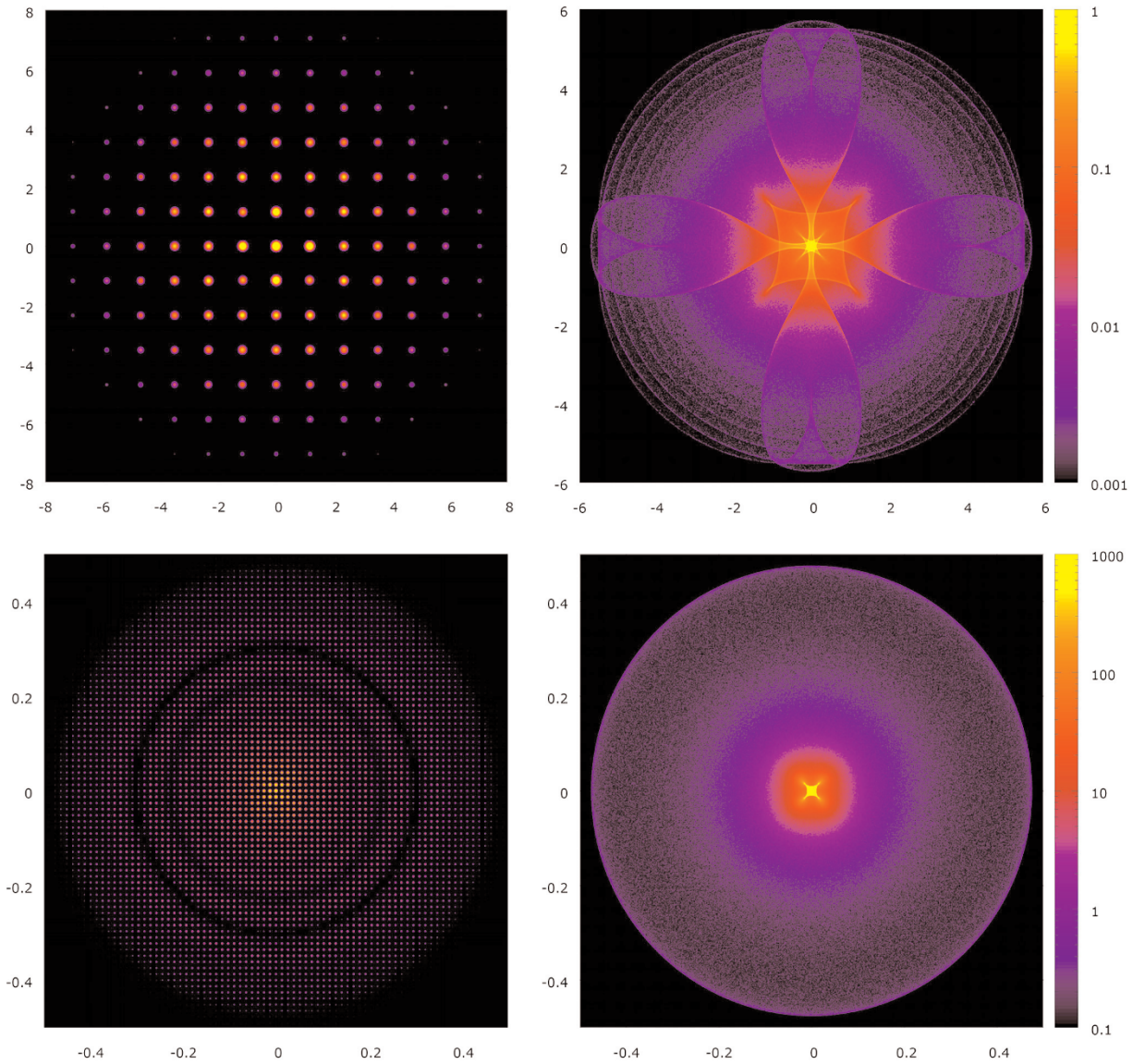

Fig. 2. *Simulation of quantum (left) and classical (right) scattering pictures of electrons in 1000 Å Si ⟨100⟩ crystal. Kinetic energies of particles are 5 MeV (above) and 500 MeV (below), from top to bottom.*

The captions at x and y axes express scattering angles in mrad, color scale expresses the value $\frac{1}{S}\frac{d\sigma}{d\vartheta^2}$ *in* mrad^{-2}

$D>>b/\Delta\vartheta$ from the target, where b is the beam transversal size and $\Delta\vartheta$ is the required angular resolution. The influence of these and similar factors onto the scattering characteristics is, however, not analyzed in the present paper.

It is noteworthy that in the range of thicknesses under study the quantum levels of transverse motion are still not developed (for that these levels were formed, it is necessary that the particle performs at least several oscillations inside the channel). At the same time, in this thickness range there is a substantial rearrangement of the wave packet (plane wave) falling onto the crystal, which is connected with the periodicity of the atomic strings positions. Due to this rearrangement, the formation of interference peaks in angular distributions of scattered particles is realized. It is also necessary to pay attention to the fact that with the target thickness increase the relative values of the scattering cross-section in these peaks change themselves. There is some periodicity in these intensity changes, so as their period is of the order of the length $l\sim a/\psi_c$ along the z axis, that substantially exceeds the distance a between the string atoms. This, in its turn, must result in an oscillatory dependence of electron radiation characteristics in crystal, and, moreover, such an effect is possible for single electrons. The detailed analysis of these effects, however, exceeds the frames of the present work.

Notably, the structure of the obtained quantum angular distributions of scattered particles is analogical to the structure of angular distributions of parti-

cles obtained from the electron microscopy (see, e. g. [14, 15]). The considered problem, however, deals with the particles whose energies substantially exceed those of the particles used in the electron microscopy. Moreover, in the electron microscopy the analysis of the process of particle scattering is performed on the basis of two- and many-wave formalism. In the present work such an analysis is performed on the basis of spectral method of direct numeric calculation of the wave function and the scattering amplitude. This provides new possibilities in the description of physical processes at the interaction of high energy particles with crystals, such as the analysis of transition from the quantum picture of scattering of charged particles in the crystal to the classical one, the rainbow scattering phenomenon, radiation in the transitional region of crystal thickness at which the channeling regime is still not formed, etc. This method can also be applied to the problems connected with the electron microscopy.

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