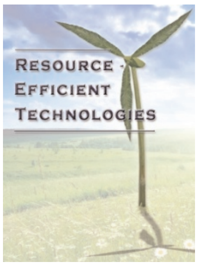
TOMSK
POLYTECHNIC
UNIVERSITY

Resource-Efficient Technologies 4 (2018) 14–18

Resource-Efficient Technologies

journal homepage: http://reffit.tech/index.php/res-eff



Research Paper

PECULIARITIES OF ELECTROMAGNETIC OSCILLATIONS GENERATED BY A CHARGED PARTICLE CROSSING THE PLANAR BOUNDARY BETWEEN A CONDUCTING MEDIUM AND A VACUUM

L.Sh. Grigoryan, A.H. Mkrtchyan, H.F. Khachatryan, M.L. Grigoryan, A.V. Sargsyan*, P.H. Mosoyan

Institute of Applied Problems of Physics of NAS RA,
25 Hr. Nersessian Str., 0014 Yerevan, Republic of Armenia

Abstract

The peculiarities of electromagnetic oscillations generated by a charged particle moving rectilinearly and uniformly have been studied when the particle crosses a planar boundary between a conducting medium and a vacuum perpendicular to that boundary. This study is based on the relevant exact analytical solutions of Maxwell equations, and the generalized Drude–Lorentz–Sommerfeld formula has been used for the dielectric function of conducting medium in the numerical calculations. The results of our investigation indicated that a charged particle may generate large amplitude oscillations in an electric field at frequencies wherein the dispersion phenomenon is essential and the real part of the conducting material's dielectric function is negative. The results further revealed that these oscillations are localized at the planar boundary of the conducting medium and a vacuum. The possibility of using this phenomenon to generate electromagnetic radiation at large distances from the surface of a conducting medium of finite size is also discussed.

Key words: electron, conducting medium, electromagnetic oscillations, dispersion, resonant radiation.

1. Introduction

Creating sources of monochromatic and spatially and temporally controllable electromagnetic radiation with high intensity and power has become one of the most exciting and compelling areas of research in modern physics [1–8].

The influence of matter on electromagnetic processes covers a wide range of phenomena that have several important practical applications [1–8], including a large group of phenomena associated with the influence of matter on the radiation of charged particles, such as transition radiation (TR) [1–3], channeling radiation [4, 5], and Cherenkov radiation (ChR) [6].

Interesting phenomena have been observed in a periodic medium [8]. The case of a charged particle moving in a waveguide completely filled with a periodic, particularly a layered medium, was investigated [9]. The radiation from a charged particle moving uniformly along the axis of a waveguide filled with a semi-infinite layered dielectric material was studied [10]. The radiation from relativistic electron moving uniformly along an infinitely long waveguide containing a finite-sized layered dielectric material was investigated [11] and a visual interpretation of the results was provided.

---

* Corresponding author. Institute of Applied Problems of Physics of NAS RA, 25 Hr. Nersessian Str., 0014 Yerevan, Republic of Armenia. E-mail address: anush.sarg@gmail.com

Peer review under responsibility of Tomsk Polytechnic University.
https://doi.org/10.18799/24056537/2018/4/215

The interfaces between media can be used to monitor the flow of radiation emitted in various systems. The interference between TR, ChR, and synchrotron radiation induced by planar, spherical, or cylindrical boundaries resulted in interesting effects [10–12].

Investigations of radiation from a charged particle rotating along an equatorial orbit around or within a dielectric ball revealed high narrow peaks in the spectral distribution of the number of quanta emitted to outer space ([12] and the references cited therein). The radiated energy in the vicinity of these peaks exceeded the corresponding value by several orders of magnitude associated with a homogeneous and unbounded medium. A similar (but weaker than in the previous case) phenomenon occurs in a cylindrically symmetric medium [13–19]. For example, the radiation emitted from (i) a longitudinal charged oscillator moving with a constant drift velocity along a cylindrical axis and (ii) a charged particle moving along a circle around a dielectric cylinder or along a helical orbit inside a cylinder have also been investigated [13–19]. High narrow peaks have been theoretically revealed in the spectral-angular distribution for the number of radiated quanta if the Cherenkov condition is satisfied for permittivity of a cylinder and the particle speed. The radiated energy in the vicinity of these peaks exceeds the corresponding value for a homogeneous medium by many times.

On a media interface, localized surface waves (SWs) can be generated if the source of the field moves near this interface and the phase velocity of the SWs can be many times less than the phase velocity of the volume waves. This particular phenomenon can have important practical applications, and consequently, this paper focuses on this subject.

The features of electromagnetic field oscillations generated by a charged particle crossing the planar boundary of a semi-infinite medium and a vacuum (along the normal to this boundary) have been investigated in previous studies [1–3]. The novelties presented herein include the consideration of frequencies for which the dispersion phenomenon is particularly significant and that the real part of the dielectric function of the conducting semi-infinite medium is negative.

2. Formulation of the problem

First, we considered a charged particle moving rectilinearly and uniformly that crosses the planar boundary of a conducting semi-infinite medium and a vacuum (along the normal to this boundary) (see Fig. 1, $\varepsilon_2 = \mu_2 = 1$).

The electromagnetic field generated by a charged particle passing through the planar boundary of a conducting semi-infinite medium and a vacuum has been previously investigated in several studies ([1–3] and the references cited therein). Formulas that determine the spectral distribution of the energy of radiation from the charged particle in the forward and backward directions have not been presented herein for brevity [2].

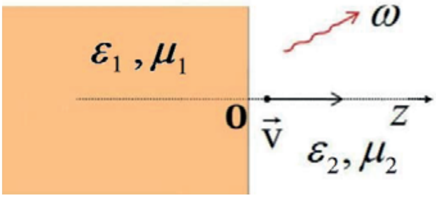


Fig. 1. A charged particle passing through the planar boundary of a conducting semi-infinite medium and a vacuum ($\varepsilon_2 = \mu_2 = 1$)

In this paper, the dielectric function of the matter of the semi-infinite medium will be described by the generalized Drude–Lorentz–Sommerfeld formula for the first time, as expressed by equation (1):

$$\varepsilon_1(\omega) = \varepsilon_0 - \frac{\omega_p^2}{\omega^2 + i\gamma\omega} = \varepsilon_1'(\omega) + i \cdot \varepsilon_1''(\omega). \tag{1}$$

This expression satisfactorily describes the dielectric function of noble metals. Taking gold as an example material [20],

$$\varepsilon_0^{Au} = 9.84, \ \hbar\omega_p^{Au} = 9.01 eV, \ \hbar\gamma^{Au} = 0.072 eV. \tag{2}$$

In equation (1), the parameter $\varepsilon_0 > 1$ describes the contribution from the bound electrons, ω_p is the plasma frequency that is associated with an effective concentration of free electrons, and γ is the phenomenological constant of electron motion attenuation (i.e., the damping constant).

The magnetic permeability of the material used as the semi-infinite medium μ_1 is assumed to be constant (for numerical calculations: $\mu_1 = 1$).

We have considered the electromagnetic oscillations in the frequency range for which

$$\varepsilon_1'(\omega) < 0. \tag{3}$$

In such cases, the electromagnetic oscillations generated inside the semi-infinite medium must be localized. In addition, we examined the projection of the generated electric field strength $\vec{E}$ on the direction in which the charged particle is moving:

$$(\vec{v} \cdot \vec{E}) / v \equiv E. \tag{4}$$

Next, the Fourier image was considered with respect to time and two Cartesian coordinates x, y parallel to the planar boundary of the semi-infinite medium and a vacuum:

$$E(\vec{r},t) = \iiint E(\omega,\vec{\chi};z)\exp[i(\vec{\chi}\vec{\rho} - \omega t)]d\omega d\vec{\chi},$$
$$\vec{\chi} = (\chi_1, \chi_2), \ \vec{\rho} = (x, y). \tag{5}$$

Referring to [1–3], this Fourier image may be expressed by:

$$E(\omega,\vec{\chi};z) = E_q(\omega,\vec{\chi};z) + E_{mb}(\omega,\vec{\chi};z), \tag{6}$$

where the first summand describes the proper field of the charge, and the second summand represents the influence of the semi-infinite medium boundary (*mb*) on the charged particle's electric field.

In a vacuum, the following expression is used:

$$E_{mb}(\omega,\vec{\chi};z)=\frac{iq}{2\pi^2\omega}a_2\exp(i\omega\tau_2 z/c)\ \text{ for } z=0. \tag{7}$$

Accordingly, inside a semi-infinite medium,

$$E_{mb}(\omega,\vec{\chi};z)=\frac{iq}{2\pi^2\omega}a_1\exp(-i\omega\tau_1 z/c)\ \text{ for } z<0, \tag{8}$$

where the dimensionless multiplier a_1 is defined by the following expression [2]:

$$a_2=-\frac{\mathrm{v}}{c}\frac{\chi^2c^2}{\omega^2}\frac{1}{\varepsilon_1\tau_2+\varepsilon_2\tau_1}\times$$

$$\times\left(\frac{1+\frac{\mathrm{v}}{c}\tau_1}{1-\frac{\mathrm{v}^2}{c^2}\varepsilon_1\mu_1+\frac{\chi^2\mathrm{v}^2}{\omega^2}}-\frac{\frac{\varepsilon_1}{\varepsilon_2}+\frac{\mathrm{v}}{c}\tau_1}{1-\frac{\mathrm{v}^2}{c^2}\varepsilon_2\mu_2+\frac{\chi^2\mathrm{v}^2}{\omega^2}}\right), \tag{9}$$

where

$$\tau_1=\sqrt{\varepsilon_1\mu_1-\chi^2c^2/\omega^2}\ \text{ and }\ \tau_2=\sqrt{\varepsilon_2\mu_2-\chi^2c^2/\omega^2}.$$

In equation (8), a_1 is not presented for brevity [3].

When $\chi c/\omega<1$, equation (7) describes the electromagnetic wave that propagates away from a semi-infinite medium along the direction of $z>0$, and for $\chi c/\omega>1$, it describes an electromagnetic field localized near the surface of the semi-infinite medium.

We next examined the phenomenon of $E_{mb}(\omega,\vec{\chi};z)$ localization at the surface $z=0$ of a semi-infinite medium in greater detail.

3. Numerical results

Numerical calculations were performed for a semi-infinite medium made of gold. The energy of the charged particle (electron) was assumed to be 2 MeV.

Fig. 2 presents a plot of the dependence of the function $|a_2(\chi,\omega,z=0)|^2$ on the cyclic frequency ω and the wave number χ of the two-dimensional harmonic oscillations $\exp[i(\vec{\chi}\vec{\rho}-\omega t)]$ appearing in the Fourier expansion (5) on the surface of the semi-infinite medium ($z=0$).

From the data presented in Fig. 2, the surface described by the values of the two-dimensional function $|a_2(\chi,\omega,z=0)|^2$ indicates that there is a branch of maxima where $|a_2(\chi,\omega,z=0)|^2>>1$, and these maxima are located in the region ; therefore, in the expansion (equation (5)), they correspond to oscillations $E(\omega,\vec{\chi};z)\exp[i(\vec{\chi}\vec{\rho}-\omega t)]$ with high amplitude $E(\omega,\vec{\chi};z)$ (see equation (7)), which are localized near the medium surface $z=0$.

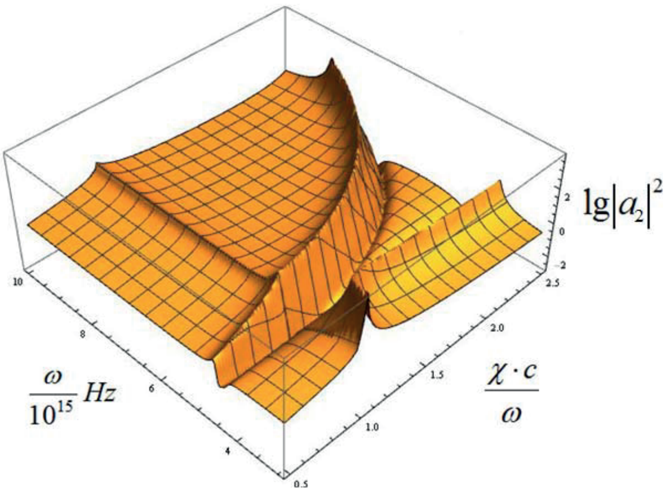


Fig. 2. The dependence of function $|a_2(\chi,\omega,z=0)|^2$ on cyclic frequency ω and wave number χ of two-dimensional harmonic oscillations $\exp[i(\vec{\chi}\vec{\rho}-\omega t)]$ appearing in the Fourier expansion (equation (5)) on the surface of the semi-infinite medium ($z=0$) generated by a 2-MeV electron crossing a planar boundary between a gold medium and a vacuum

* * *

This large amplitude of localized oscillations in the Fourier expansion of the electric field strength is not represented by the corresponding maximum in the spectral distribution of the radiation emitted by the particle (see the dashed curve in Fig. 4). This circumstance appears to be due to the previously mentioned large amplitude oscillations that are localized on the plane associated with the infinite surface of the conducting semi-infinite medium.

The question we then sought to answer was whether there are any cases where large amplitude oscillations in the electric field strength at certain frequencies are localized at the surface of a conducting medium and accompanied by a peak in the spectral distribution of the radiation emitted by a particle.

This appears possible when the surface of a conducting medium is characterized by a finite curvature of the wavelength order of the generated radiation; for example, a charged particle crossing a conducting disk in an empty space or inside a cylindrical waveguide (see Fig. 3*a*).

To confirm this hypothesis, in [21], the radiation from a charged particle moving rectilinearly and uniformly was investigated assuming that the particle passes through the center of a conducting ball.

In [21], there were peaks in the emission spectrum $W(\omega)$ of the particle near definite «resonant frequencies» (with the wavelength of the order of the ball radius). The values of the spectral density $W(\omega)$ of ra-

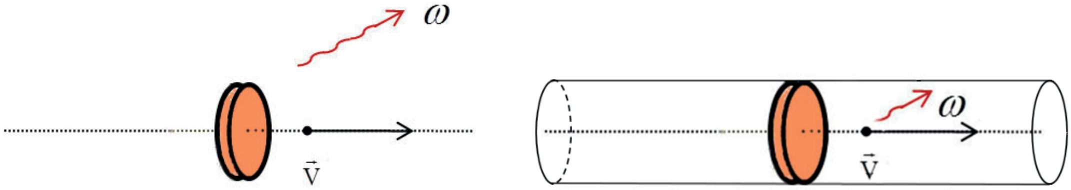


Fig. 3a. *A charged particle passing through a conducting disk in an empty space, or inside a cylindrical waveguide*

diated energy at these resonant frequencies may be many times larger than those values at adjacent frequencies (see continuous curve in Fig. 4). The peak in the spectrum of the particle radiation disappears if the ball is replaced by a semi-infinite medium made of the same conducting material (see Fig. 4).

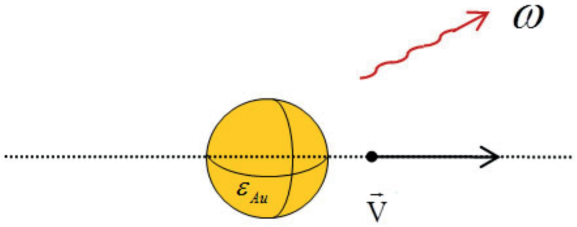


Fig. 3b. *A charged particle passing through the center of a conducting ball*

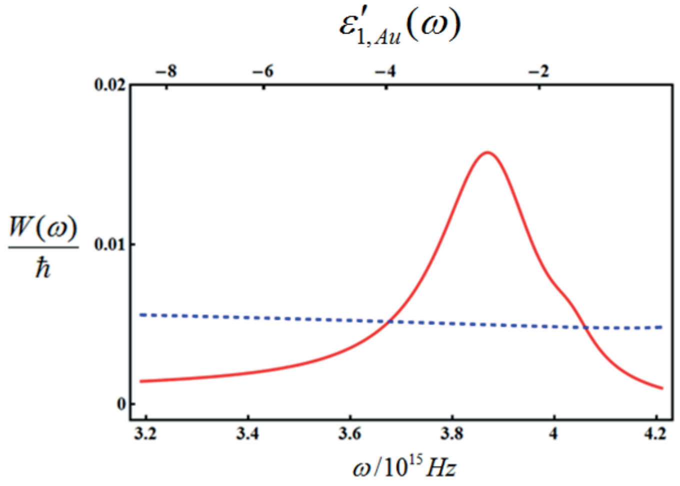


Fig. 4. *Spectral distribution of radiation from an electron: passing through the gold ball with 100-nm radius (continues line) [21] and crossing a planar interface separating a conducting semi-infinite medium (made of gold) with vacuum, along the normal to its boundary (dashed line) according to [2, 3] in the case expressed by equation (1)*

The energy of the electron is 2 MeV.

The continuous curve in Fig. 4 describes the spectral distribution of the radiation energy $\int W(\omega)d\omega$ from an electron passing through the gold ball with a radius of 100 nm. The dashed curve in Fig. 4 describes the radiation of that same electron crossing a planar boundary, which separates the conducting semi-infinite medium with vacuum, along the normal to its boundary.

The volume of the ball is much smaller than that of the semi-infinite medium; therefore, it is natural to expect that the radiation in the ball scenario should be much weaker than that associated with the semi-infinite medium. The path of the curve in Fig. 4 confirms this circumstance with the exception of a narrow frequency region near the resonant frequency $3.87\cdot10^{15}$ Hz. We next determined why the spectral density of the radiation in the ball case is larger than that for the semi-infinite medium in the vicinity of a resonant frequency.

The key to answering this question is shown in the upper part of Fig. 4, where the values of $\varepsilon'_{1,Au}$ are presented on the abscissa. As previously indicated (see equation (3)) and supported by the presented data, the real part of the dielectric function of the conducting ball at the resonant frequency assumes a negative value: $\varepsilon'_{1,Au} = -2.7 < 0$.

In such a case, the dispersion equation forbids the propagation of electromagnetic waves in the ball material along at least one of the three independent directions in space. However, it is possible to propagate electromagnetic oscillations that are localized at the boundary between the substance and the vacuum (SWs). It is clear that SWs are generated in the case of the ball and the semi-infinite medium but with one important difference. In the case of the semi-infinite medium, the vacuum interface is infinite; in the case of the ball, it is finite and enclosed. In the latter case, at most frequencies, a destructive superposition of electromagnetic oscillations on each other occurs, which leads to a decrease in their resulting amplitude. However, at certain frequencies (e.g., natural frequencies of the ball), there is a constructive superposition of electromagnetic oscillations on each other, which significantly increases their resulting amplitude.

These SWs behave like resonant radiation far from the ball, in contrast to the case of conducting semi-infinite medium that has a planar boundary with a vacuum.

4. Conclusions

The peculiarities of electromagnetic oscillations generated by a charged particle moving rectilinearly and uniformly have been studied when the particle crosses a planar boundary between a conducting medium and a vacuum perpendicular to that boundary based on the relevant exact solutions of Maxwell equations [1–3]. The generalized Drude–Lorentz–Sommerfeld formula (1) for the dielectric function of a conducting medium has been used in our numerical calculations.

The numerical calculations were performed for an electron with an assumed energy of 2 MeV crossing the planar boundary between a gold conducting medium and a vacuum. We then considered the oscillations of an electromagnetic field with frequencies that are (a) less than the frequency of plasma oscillations of free charge carriers in the semi-infinite medium for which (b) the phenomenon of dispersion is significant, and (c) the real part of the dielectric function of the semi-infinite medium is negative (see equations (1) and (3)).

Results indicated that on the surface of the semi-infinite medium, the relativistic charged particle generates large amplitude electromagnetic oscillations in the partial Fourier expansion (equation (5)). At the same time, as the distance from the medium surface increases, the amplitude of these oscillations decreases exponentially.

Thus, a relativistic charged particle that crosses the planar boundary between a conducting medium and a vacuum perpendicular to that boundary generates plasma oscillations in a high-amplitude electromagnetic field at the surface of that conducting medium.

This phenomenon may be used to generate resonant radiation emitted by a relativistic charged particle passing through a conducting medium of finite size (see Figs. 3*a*, 3*b* and 4).

Acknowledgement

The authors would like to express appreciation to Ms. Gohar Hovhannesyan for helping in performing the numerical calculations.

References

[1] Ter-Mikaelyan M.L. *Influence of the Medium on Electromagnetic Processes at High Energies*. Yerevan, AN Arm. SSR Press, 1969, 460 p. In Russian.

[2] Garibian G.M., Yang C. *X-Ray Transition Radiation*. Yerevan, AN Arm. SSR Press, 320 p., 1983. In Russian.

[3] Ginzburg V.L., Tsytovich V.N. *Transition Radiation and Transition Scattering*. Bristol, Adam Hilger, 1990, 433 p.

[4] Kumakhov M.A. *Radiation of Channeled Particles in Crystals*. Moscow, Energoatomizdat, 1986, 161 p. In Russian.

[5] Bazylev V.A., Zhevago N.K. *Emission from fast particles in matter and in external fields*. Nauka, Moscow, 1987, 272 p. In Russian.

[6] Zrelov V.P. *Vavilov-Cherenkov Radiation and its Applications in High Energy Physics*. Moscow, Atomizdat, vols. 1–2, 1968. In Russian.

[7] Rullhusen P., Artru X., Dhez P. *Novel Radiation Sources Using Relativistic Electrons*. Singapore, World Scientific, 1998, 212 p. doi: org/10.1142/3398.

[8] Potylitsin A.P. *Electromagnetic Radiation of Electrons in Periodic Structures*. Berlin Springer, 2011, 508 p. doi: 10.1007/978–3-642–19248–7.

[9] Fainberg Y.B., Khizhniak N.A. Energy losses of charged particle crossing sandwiched layer dielectric. *Sov. Phys. JETP*, 1957, vol. 5, p. 720.

[10] Grigoryan L.Sh., Khachatryan H.F., Arzumanyan S.R. Self-amplified Cherenkov radiation from relativistic particles in layered dielectric-filled waveguide. *IL Nuovo Cimento C*, 2011, vol. 34, no. 4, pp. 317–325. doi: 10.1393/ncc/i2011–10951–5.

[11] Grigoryan L.Sh., Mkrtchyan A.R., Khachatryan H.F., Arzumanyan S.R., Wagner W. Self-amplified Cherenkov radiation from a relativistic electron in a waveguide partially filled with a laminated material. *Journal of Physics: Conference Series*, 2012, vol. 357, article number 012004. doi: 10.1088/1742–6596/357/1/012004.

[12] Grigoryan L.Sh., Khachatryan H.F., Arzumanyan S.R., Grigoryan M.L. High power Cherenkov radiation from a relativistic particle rotating around a dielectric ball. *Nuclear Instruments and Methods in Physics Research, Section B*, 2006, vol. 252, no. 1, pp. 50–56. doi: 10.1016/j.nimb.2006.05.024.

[13] Kotanjyan A.S., Saharian A.A. *Izv. Nats. Akad. Nauk Arm., Fiz.* (Engl. Transl.: *J. Contemp. Phys.*). 2002, vol. 37, p. 135.

[14] Saharian A.A., Kotanjyan A.S. *Izv. Nats. Akad. Nauk Arm., Fiz.* (Engl. Transl.: *J. Contemp. Phys.*), 2003, vol. 38, p. 288.

[15] Saharian A.A., Kotanjyan A.S. Radiation from an oscillator uniformly moving along the axis of a dielectric cylinder. *Nuclear Instruments and Methods in Physics Research, Section B,* 2004, vol. 226, no. 3, pp. 351–364. doi: 10.1016/j.nimb.2004.06.036

[16] Kotanjyan A.S., Khachtaryan H.F., Petrosyan A.V., Saharian A.A. *Izv. Nats. Akad. Nauk Arm., Fiz.* (Engl. Transl.: *J. Contemp. Phys.*), 2000, vol. 35, p. 115.

[17] Kotanjyan A.S., Saharian A.A. Radiation from a charge circulating inside a waveguide with dielectric filling. *Mod. Phys. Lett. A,* 2002, vol. 17, no. 20, pp. 1323–1333. doi: 10.1142/S0217732302007272.

[18] Kotanjyan A.S. Radiation from a charged particle rotating inside a dielectric cylinder. *Nuclear Instruments and Methods in Physics Research, Section B*, 2003, vol. 201, no. 1, pp. 3–8. doi: 10.1016/S0168–583X (02)01338–1

[19] Saharian A.A., Kotanjyan A.S. Synchrotron radiation from a charge moving along a helical orbit inside a dielectric cylinder. *Journal of Physics A: Mathematical and General*, 2005, vol. 38, no. 19, pp. 4275–4292. doi: 10.1088/0305–4470/38/19/016

[20] Kolwas K., Derkachova A. Damping rates of surface plasmons for particles of size from nano- to micrometers; reduction of the nonradiative decay. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 2013, vol. 114, pp. 45–55. doi: 10.1016/j.jqsrt.2012.08.007

[21] Grigoryan L.Sh., Mkrtchyan A.H., Khachatryan H.F. *Proceedings of the International Conference on Electron, Positron, Neutron and X-Ray Scattering under External Influences*, Armenia, September 14–20, 2015, Yerevan 2016, pp. 47–52.

Received 10.12.2018