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Research paper

Improved controller design for two-input-two-output (TITO) unstable processes

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Abstract

Controller design for unstable processes is relatively difficult when compared to stable processes. The complexity increases further for multivariable unstable processes. In this work, simplified tuning rules are proposed to design PID controller for unstable multivariable processes. Decouplers are applied to make the loops independent and diagonal elements of equivalent transfer function are used to design controllers. For this, the decoupler design procedure proposed by Hazarika and Chidambaram [10] is used. Two theoretical examples of TITO unstable processes with time delays are considered for simulation. Comparative analysis has been carried out with the recently reported methods in the literature and observed that the proposed method provides improved closed loop performances. Robustness studies are also carried out with various perturbations in the processes.

Keywords: Unstable systems; Multivariable process; Decoupler; Equivalent transfer function matrix

1. Introduction

The closed loop responses for unstable systems usually have more overshoot and settling times than those of stable systems. Several methods are available for the controller design of unstable SISO systems [1]. Recently, a review on unstable systems is given by Rao and Chidambaram [2]. Whenever a process transfer function has at least one right half plane pole, it is called as an unstable process. When there are more than one input and output, if the individual transfer functions have right half plane poles, then the corresponding processes is called as multivariable unstable process. Controller design methods for MIMO unstable systems are limited and this research has taken attention in the recent past. Interactions between the loops make the design more difficult. Performances of MIMO systems can be enhanced by the use of decouplers to counter act process interactions. But these decouplers are sensitive to changes in process and need highly accurate process models, which are difficult to find. Decoupling can be done in three ways such as ideal, inverted and simplified. For ideal decoupling inverse of the process has to be found, which gives a complicated decoupler elements. Inverted decoupling is sensitive to modeling errors. Simplified decoupling results in a simple form, but controller cannot be applied directly to this decoupled process without any model reduction method. Few methods are reported to design controllers for unstable multivariable systems. Examples of multivariable unstable process are described in [3–5]. Georgiou et al. [3] proposed a decentralized PID controller design for unstable multivariable systems using optimization method. Agamennini et al. [6] proposed a design using least squares method for time delay unstable systems with multivariable delay compensator. Govindhakannan and Chidambaram [7] applied the method of Tanttu and Lieslehto [8] method to design single stage PI controllers for unstable multivariable systems. Chandrasekhar and Chidambaram [9] proposed a simple method of designing decentralized PID controllers for unstable systems by synthesis method. Rajapandiyan et al. [10] designed a controller for multi loop stable processes based on ETF (equivalent transfer function) approximation with simplified decouplers. Recently, Dasari et al. [11] used ETF to design the controllers based on optimal H_2 IMC principles, simplified decouplers are also included and observed enhanced performance for unstable systems. Hazarika and Chidambaram [12] proposed a double loop control

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structure to decrease the overshoot such as proportional controller followed by PI controller in the outer loop based on the equivalent transfer function. They have showed that by a single loop PI control with a set point filter, the overshoot is reduced significantly and a good servo response is obtained. However, this will not improve the regulatory responses. It may be desirable to use better settings particularly with a PID controller to improve the performances of both the servo and regulatory performances. All of these methods follow a complex procedure to design the controller. In this work, simple tuning rules are developed and applied for MIMO unstable first order time delay systems.

2. Criteria of pairing

On selecting the appropriate input–output pairing, problem of loop interaction can be reduced. Relative gain array (RGA) and Niederlinski index (NI) are used to pair the manipulated and controlled variables. For stable systems, the selection of pairing corresponds to the positive values of NI and RGA, where

$$NI = \frac{\det[K]}{\prod K_{ii}} \tag{1}$$

$$RGA = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix} = K \otimes K^{-T} \tag{2}$$

Criteria of pairing for unstable systems will differ when the number of unstable open-loop poles of $G_p(s)$ is different from $\bar{G}_p(s) = \mathrm{diag}[g_{p,ii}(s)]$. Then pairing should be made in the following ways: For a $n \times n$ square system (1) with one unstable pole which exists in all elements of the process $G_p(s)$, pairing should be made such that NI is positive if n is odd and NI should be negative if n is even; (2) with P number of unstable poles are there in all elements of $G_p(s)$, pairing should be such that NI is positive if $(n - 1)P$ is even and negative if $(n - 1)P$ is odd.

3. Decoupler design

The methodology proposed by Hazarika and Chidambaram [12] is used here. Many MIMO problems can be modified so that decentralized control becomes a more viable (or attractive) option. For example, one can sometimes use a pre-compensator to turn the resultant system into a more nearly diagonal transfer function. Consider the process G_p with input U and output Y as shown in the block diagram

$$Y(s) = G_p(s)D(s)U(s)$$

$$\begin{bmatrix} g_{p,11} & g_{p,12} \\ g_{p,21} & g_{p,22} \end{bmatrix} \begin{bmatrix} 1 & d_{12} \\ d_{21} & 1 \end{bmatrix} = \begin{bmatrix} g^*_{p,11} & 0 \\ 0 & g^*_{p,22} \end{bmatrix} = G_p(s)D(s)$$

where the simplified decouplers are given by

$$d_{12}(s) = -\frac{g_{p,12}(s)}{g_{p,11}(s)} \quad d_{21}(s) = -\frac{g_{p,21}(s)}{g_{p,22}(s)} \tag{3}$$

4. Equivalent transfer function

Equivalent transfer function, diagonal elements are considered to design the controllers. If $y_{r2} = 0$ in the block diagram then

$$\frac{y_1}{u_1} = g_{p,11} - \frac{g_{p,12}g_{p,21}g_{c,2}}{1+g_{c,2}g_{p,22}} \tag{4}$$

this can be further written as

$$\frac{y_1}{u_1} = g_{p,11} - \frac{g_{p,12}g_{p,21}g_{c,2}g_{p,22}}{(1+g_{c,2}g_{p,22})g_{p,22}} \tag{5}$$

Similarly, if $y_{r1} = 0$

$$\frac{y_2}{u_2} = g_{p,22} - \frac{g_{p,21}g_{p,12}g_{c,1}g_{p,11}}{(1+g_{c,1}g_{p,11})g_{p,11}} \tag{6}$$

Two assumptions are made to simplify the above equations [13]: (1) perfect controller approximation for another

$$\frac{g_{c,i}g_{p,ii}}{1+g_{c,i}g_{p,ii}} = 1 \tag{7}$$

(2) ETFs have the same structure of corresponding open-loop model. By using the assumption of perfect controller approximation, eqs. (5) and (6) can be reduced to

$$g^{eff}_{p,11} = \frac{y_1}{u_1} = g_{p,11} - \frac{g_{p,12}g_{p,21}}{g_{p,22}} \tag{8}$$

$$g^{eff}_{p,22} = \frac{y_2}{u_2} = g_{p,22} - \frac{g_{p,12}g_{p,21}}{g_{p,11}} \tag{9}$$

Here, $g_{p,11}^{eff}$ and $g_{p,22}^{eff}$ are effective open-loop transfer functions (EOTF). These are complicated transfer function models. As EOTFs are complex they are approximated to FOPTD using Maclaurin series for constructing the controllers. For higher dimension systems EOTFs formulation is complex. Using (RGA) relative gain array, (RNGA) relative normalized gain array, and (RARTA) relative average residence time array concepts, for higher dimension systems the expression for ETF (equivalent transfer function) can be derived easily. The closed-loop responses using ETF should match with the closed-loop responses of $g_{p,11}^{eff}$ and $g_{p,22}^{eff}$ then both ETF and EOTF are said to be the same. Construction of ETF is as follows: The $K_{N,ij}$ normalized gain is defined as

$$K_N = K\Theta T_{ar} = \begin{bmatrix} K_{N,11} & K_{N,12} \\ K_{N,21} & K_{N,22} \end{bmatrix} \tag{10}$$

Here Θ represents Hadamard division, K denotes the steady state gain, and $T_{ar} = \tau_{ij} + \theta_{ij}$ is average residence time which indicates the response speed of y_i controlled variable to u_j manipulated variable. The RNGA relative normalized gain array is defined as

$$\phi = K_N \otimes K_N^{-T} == \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \tag{11}$$

where ⊗ represents Hadamard multiplication. RARTA relative average residence time array, which is defined as the ratio of loop $y_i - u_j$ average residence times, when other loops are closed and when other loops are open, is given by

$$\Gamma = \phi \Theta \Lambda = \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix} \tag{12}$$

Therefore, equivalent transfer function for an unstable system can be expressed as

$$\hat{g}_{p,ij} = \hat{k}_{p,ij} \frac{e^{-\hat{\theta}_{ij} s}}{\hat{\tau}_{ij} s - 1} \tag{13}$$

where $\hat{k}_{p,ij} = k_{p,ij} / \Lambda_{ij}$, $\hat{\tau}_{ij} = \gamma_{ij} \tau_{ij}$ and $\hat{\theta}_{ij} = \gamma_{ij} \theta_{ij}$. It should be noted that the method is applicable when EOTF = ETF.

5. Controller design

Diagonal elements of equivalent transfer function are considered to design the Controllers. Here, the diagonal element of ETF is considered as a UFOPDT process

$$G_p(s) = \frac{k_p e^{-\theta s}}{\tau_p s - 1} \tag{14}$$

IMC controller Q_C is given by $Q_C = \tilde{Q}_C F$, where F is a filter used to adjust the controller robustness

$$G_m = G_{m-} G_{m+} \quad and \quad v = v_- v_+ \tag{15}$$

where the subscript "+" refers to non-minimum phase and "–" refers to minimum phase part. The Blaschke product of G_m and v are defined as

$$b_m = \prod_{i=1}^{k} \frac{-s + p_i}{s + \overline{p}_i} \quad and \quad b_v = \prod_{i=1}^{\tilde{k}} \frac{-s + p_i}{s + \overline{p}_i} \tag{16}$$

where p_i is ith RHP pole and $\overline{p}_i$ is its conjugate. H_2 optimal controller is designed based on the above concepts by using the formula [14].

$$\tilde{Q}_C = b_m (G_{m-} b_v v_-)^{-1} \left\{ (b_m G_{m+})^{-1} b_v v_- \right\}\Big|_* \tag{17}$$

$$F = (\alpha s + 1) / (\lambda s + 1)^3 \tag{18}$$

therefore, IMC controller is given as

$$Q_C = \frac{(\tau_P s - 1)}{k_P} \left\{ \left(e^{\theta/\tau_P} - 1\right) \tau_P s + 1 \right\} \frac{(\alpha s + 1)}{(\lambda s + 1)^3} \tag{19}$$

Here, λ is tuning parameter. The value of α can be found from the internal stability condition of IMC.

Condition 1: Q_C should be stable and cancel the right half plane poles of G_m

Condition 2: $Q_C G_m$ must be stable

Condition 3: $(1 - G_m Q_C)$ at RHP poles of the process should be zero

From the design procedure followed, the first two conditions are justified and third condition is

$$(1 - Q_C G_m)\big|_{s = 1/\tau_P} = 0 \tag{20}$$

Substituting Q_C, the value of α is

$$\alpha = \left\{ (\lambda/\tau_P)^2 + 3(\lambda/\tau_P) + 3 \right\} \lambda \tag{21}$$

Now, IMC controller is converted into a unity feedback controller G_C as

$$G_C = \frac{Q_C}{1 - Q_C G_m} \tag{22}$$

Substituting Q_c and G_m in above equation, we get

$$G_C = \frac{\left\{ \left(e^{\theta/\tau_P} - 1\right) \tau_P s + 1 \right\} (\alpha s + 1)(\tau_P s - 1)}{k_P \left[(\lambda s + 1)^3 - \left\{ \left(e^{\theta/\tau_P} - 1\right) \tau_P s + 1 \right\} (\alpha s + 1) e^{-\theta s} \right]} \tag{23}$$

This expression can be simplified into a PID controller using Maclaurin series or Laurent series. Let us consider $J(s) = sG_c(s)$. Now, expand J(s) by using Maclaurin series expansion to get the controller G_c as

$$G_c(s) = \frac{1}{s} \left(J(0) + J'(0)s + \frac{J''(0)}{2!} s^2 + \ldots.. \right) \tag{24}$$

General form of a PID controller is

$$G_c(s) = k_c \left(1 + \frac{1}{\tau_i s} + \tau_d s \right) \tag{25}$$

The PID controller parameters are obtained by comparing the above two expressions

$$k_c = J'(0), \ \tau_i = J'(0)/J(0) \text{ and } \tau_d = J''(0)/2J'(0) \tag{26}$$

where

$$J(0) = 1 / p_m(0) D(0)$$

$$J'(0) = -[p'_m(0) D(0) + p_m(0) D'(0)] / [p_m(0) D(0)]^2$$

$$J''(0) = J'(0) \left[\frac{p''_m(0) D(0) + 2p'_m(0) D'(0) + p_m(0) D''(0)}{p'_m(0) D(0) + p_m(0) D'(0)} + 2\frac{J'(0)}{J(0)} \right]$$

$$D(0) = 3\lambda - p'_A(0); \quad D'(0) = [6\lambda^2 - p''_A(0)]/2;$$
$$D''(0) = [6\lambda^3 - p'''_A(0)]/3$$

$$p'_A(0) = \alpha - \theta + \tau_p x \qquad p''_A(0) = \theta^2 - 2\alpha\theta + 2\tau_p x(\alpha - \theta)$$

$$p'''_A(0) = -\theta^3 + 3\alpha\theta^2 + 3\tau_p x \theta^2 - 6\alpha\theta\tau_p x;$$
$$p'_m(0) = -k_p(\tau_p - \alpha - \tau_p x)$$

$$p_m(0) = -k_p; \quad p''_m(0) = -k_p\left(2\tau_p x(\tau_p - \alpha) + 2\alpha\tau_p + 2(\tau_p - \alpha - \tau_p x)^2\right); \quad x = e^{\theta/\tau_p} - 1$$

Using controller parameters substitution and simplification, we can obtain expressions for controller parameters as a function of λ/τ_p and θ/τ_p which are very complex. Each expression for k_c, τ_i, τ_d contains terms which are complex and are not operator friendly. Simple tuning rules are always preferred for all practical applications in industries. In this work, simple expressions developed by the authors are used [15]. The simple tuning rules are

$$k_c k_p = a_1\left(\frac{\lambda}{\tau_p}\right)^{b_1} + c_1 \quad (27)$$

$$\frac{\tau_i}{\tau_p} = a_2\left(\frac{\lambda}{\tau_p}\right)^{b_2} + c_2 \quad (28)$$

$$\frac{\tau_D}{\tau_p} = a_3\left(\frac{\lambda}{\tau_p}\right)^{b_3} + c_3 \quad (29)$$

where $a_1, b_1, c_1, a_2, b_2, c_2, a_3, b_3, c_3$ are coefficients which change for each ratio of time delay to time constant in the range of 0.1 to 1.2. The values of these coefficients are shown in Table 1. Coefficients for intermediate ratios of time delay to time constant, which are not given in the Table 1, can be obtained by using simple interpolation techniques.

It is well known that the tuning parameters should be selected in such a way that the resulting controllers give both nominal as well as robust performance. In IMC method, a lesser value for tuning parameters gives good nominal performance. Higher value of these tuning parameters gives robust control performance with compromise on nominal responses. Hence there is a tradeoff in selecting the tuning parameter values. After conducting many simulation studies on different processes, the recommended range for the tuning parameter is selected as $\lambda = 0.5\theta - 2\theta$. If the closed loop responses are not acceptable within this range, other values can be selected.

6. Simulation results

Example 1: Consider a TITO process [12]

$$G_p(s) = \begin{bmatrix} \frac{1.6e^{-s}}{-2.6s+1} & \frac{0.6e^{-1.5s}}{2.5s+1} \\ \frac{0.7e^{-1.5s}}{3s+1} & \frac{1.7e^{-s}}{-2.2s+1} \end{bmatrix}$$

To find out the pairing selection for this system RGA and NI are calculated

$$\mathrm{K} = \begin{bmatrix} 1.6 & 0.6 \\ 0.7 & 1.7 \end{bmatrix} \mathrm{RGA} = \begin{bmatrix} 1.1826 & -0.1826 \\ -0.1826 & 1.1826 \end{bmatrix}$$

NI = 0.6969

As the number of open loop unstable poles is same for both $G_p(s)$ and $G_p(s) = \mathrm{diag}[g_{p,ii}(s)]$, the criterion for pairing is to get a positive value for NI. Since the calculated NI is positive, pairing can be kept as it is. The simplified decouplers are designed as previously described (Fig. 1)

$$D(s) = \begin{bmatrix} 1 & \frac{(7.8s-3)e^{-0.5s}}{20s+8} \\ \frac{(15.4s-7)e^{-0.5s}}{51s+17} & 1 \end{bmatrix}$$

To find the controller parameters, equivalent transfer function can be evaluated using the following quantities such as average residence time array, normalized gain array, relative normalized gain array (RNGA) and relative average residence time array (RARTA)

$$T_{ar} = \begin{bmatrix} 3.6 & 4 \\ 4.5 & 3.2 \end{bmatrix} K_N = \begin{bmatrix} 0.444 & 0.15 \\ 0.1556 & 0.5313 \end{bmatrix}$$

$$\phi = \begin{bmatrix} 1.1097 & -0.1097 \\ -0.1097 & 1.1097 \end{bmatrix} \Gamma = \begin{bmatrix} 0.9383 & 0.6005 \\ 0.6005 & 0.9383 \end{bmatrix}$$

Table 1
Coefficients for controller parameters with different θ/τ_p values.

θ/τ_p	$k_c k_p = a_1(\lambda/\tau_p)^{b_1} + c_1$			$\frac{\tau_I}{\tau_P} = a_2(\lambda/\tau_p)^{b_2} + c_2$			$\frac{\tau_D}{\tau_P} = a_3(\lambda/\tau_p)^{b_3} + c_3$		
	a_1	b_1	c_1	a_2	b_2	c_2	a_3	b_3	c_3
0.1	1.834	−0.6529	0.3804	2.733	2.682	7.256	−0.5576	0.2843	0.3743
0.2	1.99	−0.519	0.1901	2.733	2.682	7.348	−0.4616	0.3202	0.3656
0.3	2.005	−0.4427	0.09072	2.734	2.682	7.437	−0.3814	0.3564	0.3674
0.4	1.988	−0.3888	0.0235	2.734	2.682	7.526	−0.3175	0.3905	0.382
0.5	1.963	−0.3468	−0.02819	2.734	2.682	7.617	−0.2673	0.4213	0.4078
0.6	1.935	−0.3123	−0.07073	2.734	2.682	7.71	−0.2287	0.4469	0.4437
0.7	1.908	−0.2831	−0.1069	2.734	2.682	7.808	−0.1998	0.4657	0.4886
0.8	1.88	−0.2579	−0.1377	2.733	2.682	7.913	−0.1793	0.4761	0.5418
0.9	1.852	−0.2359	−0.1632	2.733	2.683	8.025	−0.1662	0.4765	0.6029
1	1.822	−0.2166	0.1828	2.731	2.683	8.148	−0.1593	0.4675	0.6715
1.1	1.788	−0.1999	−0.1951	2.729	2.683	8.283	−0.1575	0.4514	0.7469
1.2	1.748	−0.1854	−0.1983	2.727	2.684	8.433	−0.1586	0.4334	0.8275

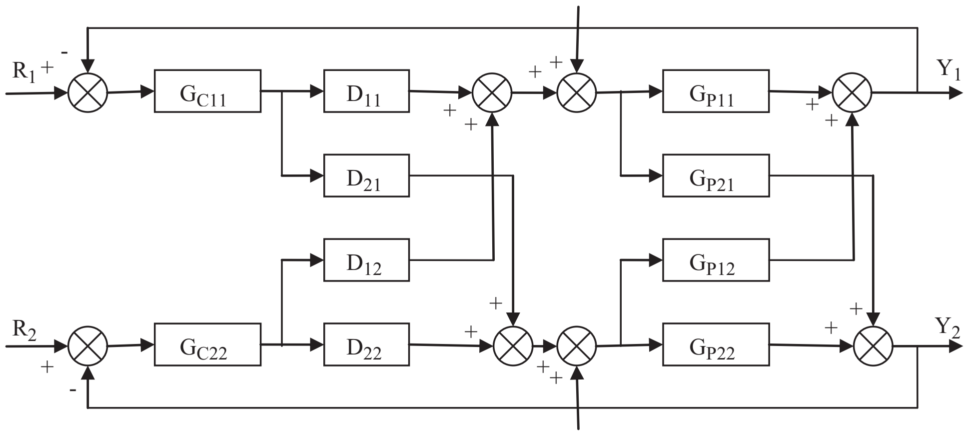

Fig. 1. Decoupled control scheme.

Using all of the above, equivalent transfer function can be developed as

$$\hat{G}_p=\begin{bmatrix}\dfrac{1.3529e^{-0.9383s}}{-2.4396s+1} & \dfrac{-3.2857e^{-0.9008s}}{1.5013s+1}\\ \dfrac{-3.8333e^{-0.9008s}}{-1.8016s+1} & \dfrac{1.4375e^{-0.9383s}}{-2.0643s+1}\end{bmatrix}$$

Controller design with lead lag filter for Dasari et al. method is given by

$$G_c=\begin{bmatrix}-0.0371\left(1+\dfrac{1}{0.6255s}+0.2346\right) & 0\\ 0 & -0.0335\left(1+\dfrac{1}{0.6255s}+0.2346\right)\end{bmatrix}$$

$$F'(s)=\begin{bmatrix}\dfrac{20.3717s+1}{0.4537s+1} & 0\\ 0 & \dfrac{20.2497s+1}{0.3965s+1}\end{bmatrix}$$

Controller design for proposed method is given by

$$G_c=\begin{bmatrix}-1.8099\left(1+\dfrac{1}{20.137s}+0.264\right) & 0\\ 0 & -1.541\left(1+\dfrac{1}{18.035s}+0.292\right)\end{bmatrix}$$

This designed controller is applied with set point weighting of magnitude 0.3 to both the loops to the process. The controllers for all methods are given in Table 2. The closed loop responses are plotted for a unit step change in the set point and disturbance separately and are compared with other methods such as Hazarika and Chidambaram [12] and Dasari et al. [11]. Fig. 2 shows the closed loop servo responses and interaction responses. Fig. 3 shows the corresponding regulatory responses. It can be observed that the proposed method provides improved performances. To analyze the robustness, perturbations of +10% in all time delays are given and the corresponding responses are shown in Fig. 4. Perturbations in other parameters are also considered and the corresponding IAE values are given in Tables 3 and 4. It can be observed that the proposed method is stable. The corresponding IAE values for all the three methods are given in Table 5 and it can be observed that the proposed method is better when compared to the other methods.

Example 2: Consider another TITO process [12]

$$G_P(S)=\begin{bmatrix}\dfrac{-1.6667e^{-s}}{-2.6s+1} & \dfrac{-1e^{-s}}{-1.6667s+1}\\ \dfrac{-0.8333e^{-s}}{-1.6667s+1} & \dfrac{-1.6667e^{-s}}{-1.6667s+1}\end{bmatrix}$$

Pairing is done based on RGA and NI

$$K=\begin{bmatrix}-1.6667 & -1\\ -0.8333 & -1.6667\end{bmatrix}$$

Table 2
Controller parameters for various methods.

	Method	Controller parameters					
		Loop1			Loop 2		
		$k_{c,11}$	$\tau_{I,11}$	$\tau_{D,11}$	$k_{c,22}$	$\tau_{I,22}$	$\tau_{D,22}$
Example 1	Proposed	−1.8099	20.137	0.264	−1.541	18.035	0.292
	Hazarika et al. (inner loop)	−1.19	0	0	−1.007	0	0
	Hazarika et al. (outer loop)	0.1902	7	0	0.155	7	0
	Dasari et al.	−0.0371	0.6255	0.2346	−0.0335	0.6255	0.2346
Example 2	Proposed	−0.695	34.89	0.244	−0.791	34.89	0.244
	Hazarika et al. (inner loop)	−0.55	0	0	−0.66	0	0
	Hazarika et al. (outer loop)	0.112	4	0	0.112	4	0
	Dasari et al.	−0.0139	0.6667	0.25	−0.0199	0.6667	0.25

Table 3
Comparison of IAE values of servo responses with perturbations in process parameters for example-1.

Perturbations	Proposed method				Hazarika method				Dasari et al. method			
	Unit step input in loop1		Unit step input in loop2		Unit step input in loop1		Unit step input in loop2		Unit step input in loop1		Unit step input in loop2	
	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2
k_p	6.14	0.02	0.02	5.07	13.96	0	0	14.10	12.42	0	0	11.90
$1.1k_p$	6.72	0.02	0.02	5.55	15.98	0	0	16.77	13.54	0	0	13.07
$0.9k_p$	5.58	0.02	0.02	4.66	11.85	0	0	11.92	11.11	0	0	10.57
τ	6.14	0.02	0.02	5.07	13.96	0	0	14.10	12.42	0	0	11.90
1.1τ	6.23	0.17	0.14	5.2	14	0.36	0.22	14.24	12.42	0.25	0.19	11.9
0.9τ	6.06	0.27	0.23	5.04	13.9	1.15	0.79	14.15	12.42	0.25	0.19	11.9
θ	6.14	0.02	0.02	5.07	13.96	0	0	14.10	12.42	0	0	11.90
1.1θ	6.12	0.19	0.17	5.1	13.96	0.57	0.39	14.3	12.42	0.04	0.03	11.9
0.9θ	6.16	0.092	0.08	5.11	13.96	0.09	0.06	14.13	9.12	0.01	0	7.74

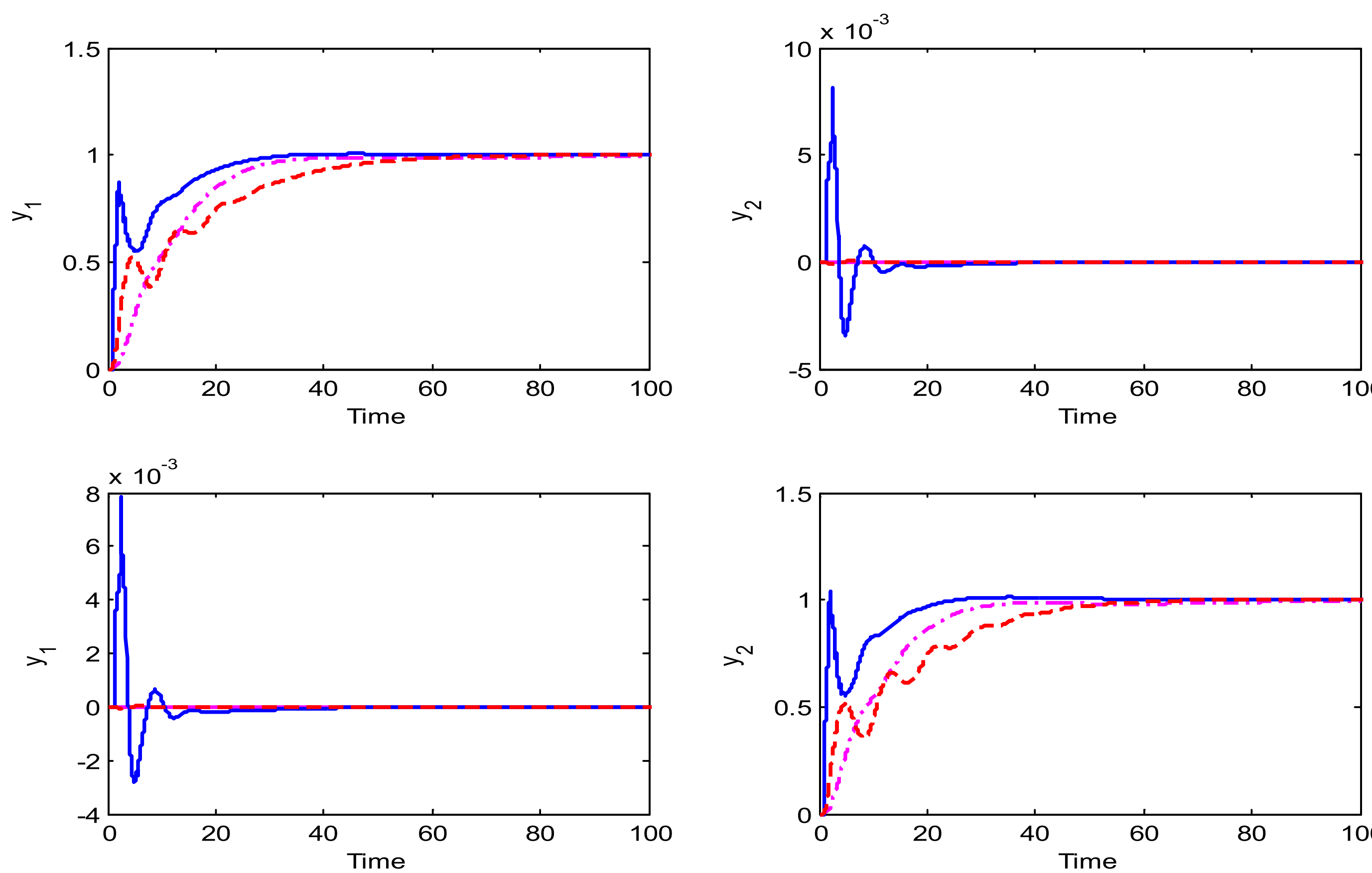

Fig. 2. Comparison of servo responses for a unit step input in y1 and y2, proposed method (solid), Hazarika method (dash) and Dasari et al. method (dash dot) for example 1.

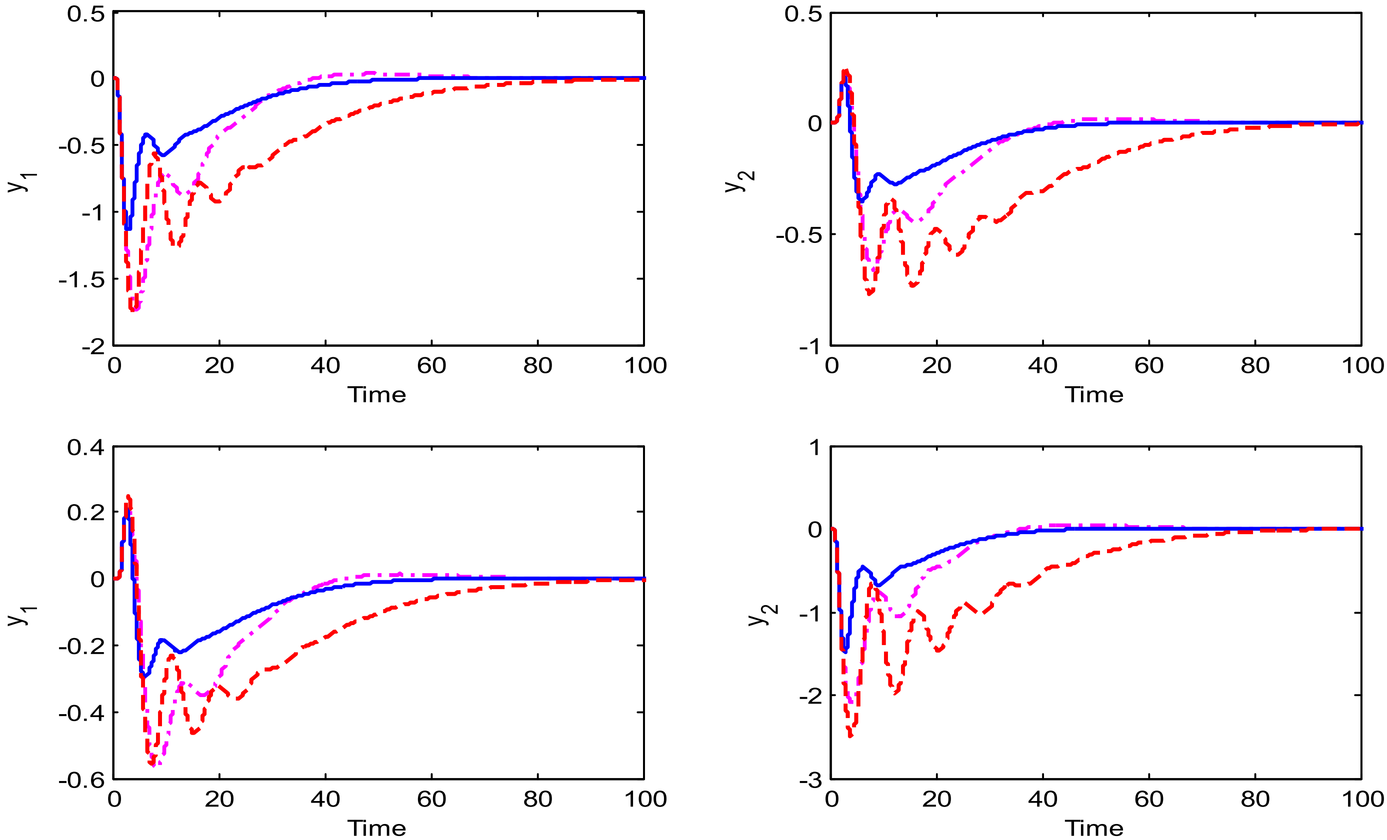

Fig. 3. Comparison of regulatory responses for a unit step input in y_1 and y_2, proposed method (solid), Hazarika method (dash) and Dasari et al (dash dot) for example 1.

$$RGA = \begin{bmatrix} 1.4283 & -0.4283 \\ -0.4283 & 1.4283 \end{bmatrix}$$

NI for this system is 0.5833. As the number of open loop unstable poles of $G_p(s)$ is different from $G_p(s) = \mathrm{diag}[g_{p,ii}(s)]$, criterion of pairing is different. Since NI calculated is positive, columns are interchanged. After interchanging

$$G_P(S) = \begin{bmatrix} \dfrac{-1e^{-s}}{-1.6667s+1} & \dfrac{-1.6667e^{-s}}{-2.6s+1} \\ \dfrac{-1.6667e^{-s}}{-1.6667s+1} & \dfrac{-0.8333e^{-s}}{-1.6667s+1} \end{bmatrix}$$

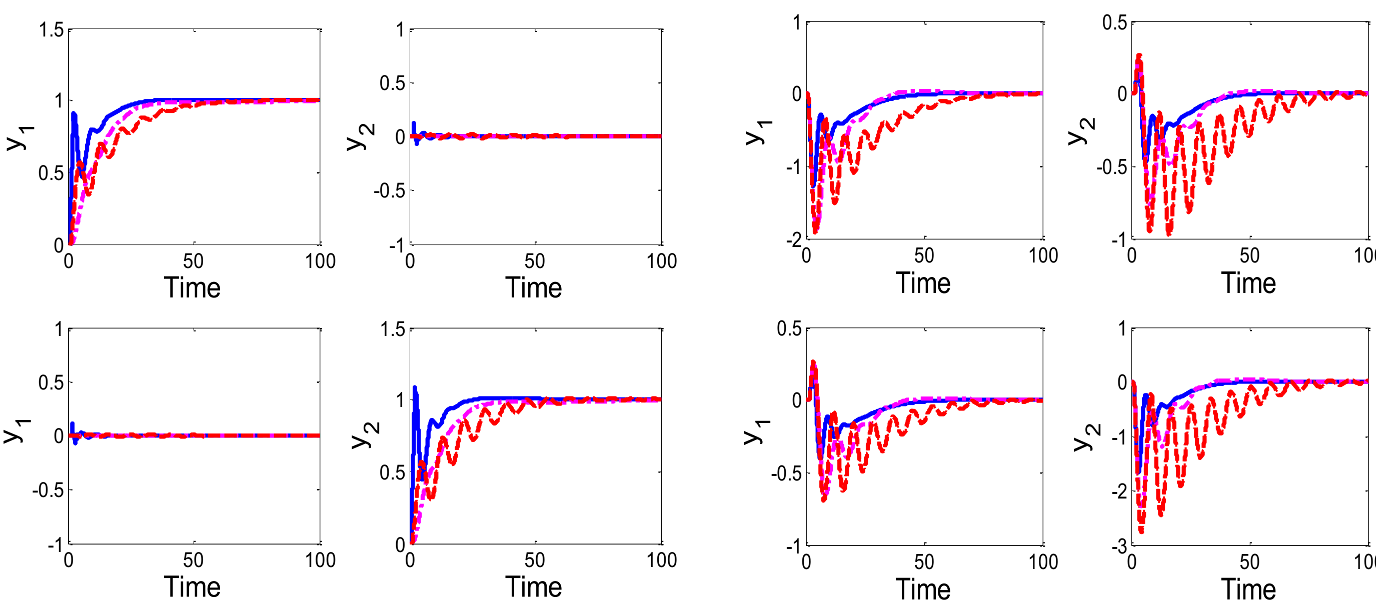

Fig. 4. Comparison of servo and regulatory responses with 10% perturbations to time delay for a unit step input in y_1 and y_2, proposed method (solid), Hazarika et al. (dash) and Dasari et al. (dash dot) for example 1.

Table 4
Comparison of IAE values of regulatory responses with perturbations for example-1.

	Proposed method				Hazarika method				Dasari et al. method			
	Unit step input in loop1		Unit step input in loop2		Unit step input in loop1		Unit step input in loop2		Unit step input in loop1		Unit step input in loop2	
	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2
k_p	13.22	6.38	5.61	14.02	36.54	22.81	14.64	53.12	21.54	10.64	8.96	24.01
$1.1k_p$	13.18	6.37	5.61	13.93	36.35	22.64	14.58	52.73	20.86	10.34	8.73	23.2
$0.9k_p$	13.33	6.43	5.63	14.2	36.8	23.22	14.7	54.33	22.78	11.19	9.39	25.48
τ	13.22	6.38	5.61	14.02	36.54	22.81	14.64	53.12	21.54	10.64	8.96	24.01
1.1τ	13.24	6.36	5.6	14.05	36.56	22.79	14.61	53.17	21.72	10.53	8.86	24.23
0.9τ	13.21	6.41	5.62	14	36.52	22.83	14.73	53.22	21.34	10.74	9.05	23.77
θ	13.22	6.38	5.61	14.02	36.54	22.81	14.64	53.12	21.54	10.64	8.96	24.01
1.1θ	13.22	6.48	5.72	14.01	36.53	23.04	14.76	53.46	21.47	10.67	9.02	23.94
0.9θ	13.23	6.3	5.51	14.04	36.55	22.71	14.53	53.14	21.62	10.61	8.91	24.11

Now the relative gain array is calculated as

$$RGA = \begin{bmatrix} -0.4283 & 1.4283 \\ 1.4283 & -0.4283 \end{bmatrix}$$

Equivalent transfer function is calculated as

$$\hat{G}_p = \begin{bmatrix} \dfrac{2.3348e^{-s}}{-1.6667s+1} & \dfrac{-1.16711e^{-s}}{-2.6s+1} \\ \dfrac{-1.16711e^{-s}}{-1.6667s+1} & \dfrac{1.9456e^{-s}}{-1.6667s+1} \end{bmatrix}$$

Decouplers are designed as

$$D(s) = \begin{bmatrix} 1 & -1.6667 \\ -2.001 & 1 \end{bmatrix}$$

Controller with lead lag filter designed by Dasari et al. [11] is

$$G_c(s) = \begin{bmatrix} -0.0139\left(1+\dfrac{1}{0.6667s}+0.25\right) & 0 \\ 0 & -0.0199\left(1+\dfrac{1}{0.6667s}+0.25\right) \end{bmatrix}$$

$$F'(s) = \begin{bmatrix} \dfrac{28.3073s+1}{0.3673s+1} & 0 \\ 0 & \dfrac{24.3746s+1}{0.3487s+1} \end{bmatrix}$$

Controller designed according to proposed method is

$$G_c(s) = \begin{bmatrix} -0.6595\left(1+\dfrac{1}{34.89s}+0.244\right) & 0 \\ 0 & -0.791\left(1+\dfrac{1}{34.89s}+0.244\right) \end{bmatrix}$$

Table 5
Comparison of IAE values for an input of unit step magnitude in set point and disturbance in each loop.

	Methods	Servo				Regulatory			
		Loop1		Loop 2		Loop1		Loop 2	
		Y_1	Y_2	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2
Example1	Proposed method	6.14	0.02	0.02	5.07	13.22	6.38	5.61	14.02
	Hazarika	13.96	0	0	14.1	36.54	22.81	14.64	53.12
	Dasari et al.	12.42	0	0	11.9	21.54	10.64	8.96	24.01
Example2	Proposed	2.18	0.01	0	2.17	22.66	37.78	37.76	18.88
	Hazarika et al.	7.91	0.01	0	7.89	27.81	46.37	46.35	23.17
	Dasari et al	8.8	0.01	0	7.74	20.58	28.75	34.29	14.37

This designed controller is applied with set point weighting of magnitude 0.3 to both the loops of the process and the closed loop responses are plotted. Fig. 5 shows the servo responses, Fig. 6 shows the regulatory responses and Fig. 7 shows the responses for perturbations. The corresponding IAE values for different perturbations are given in Tables 6 and 7. It can be observed that the proposed method is robust. The IAE values for all three methods are given in Table 5 and it can be observed that the proposed method is better compared to the other methods.

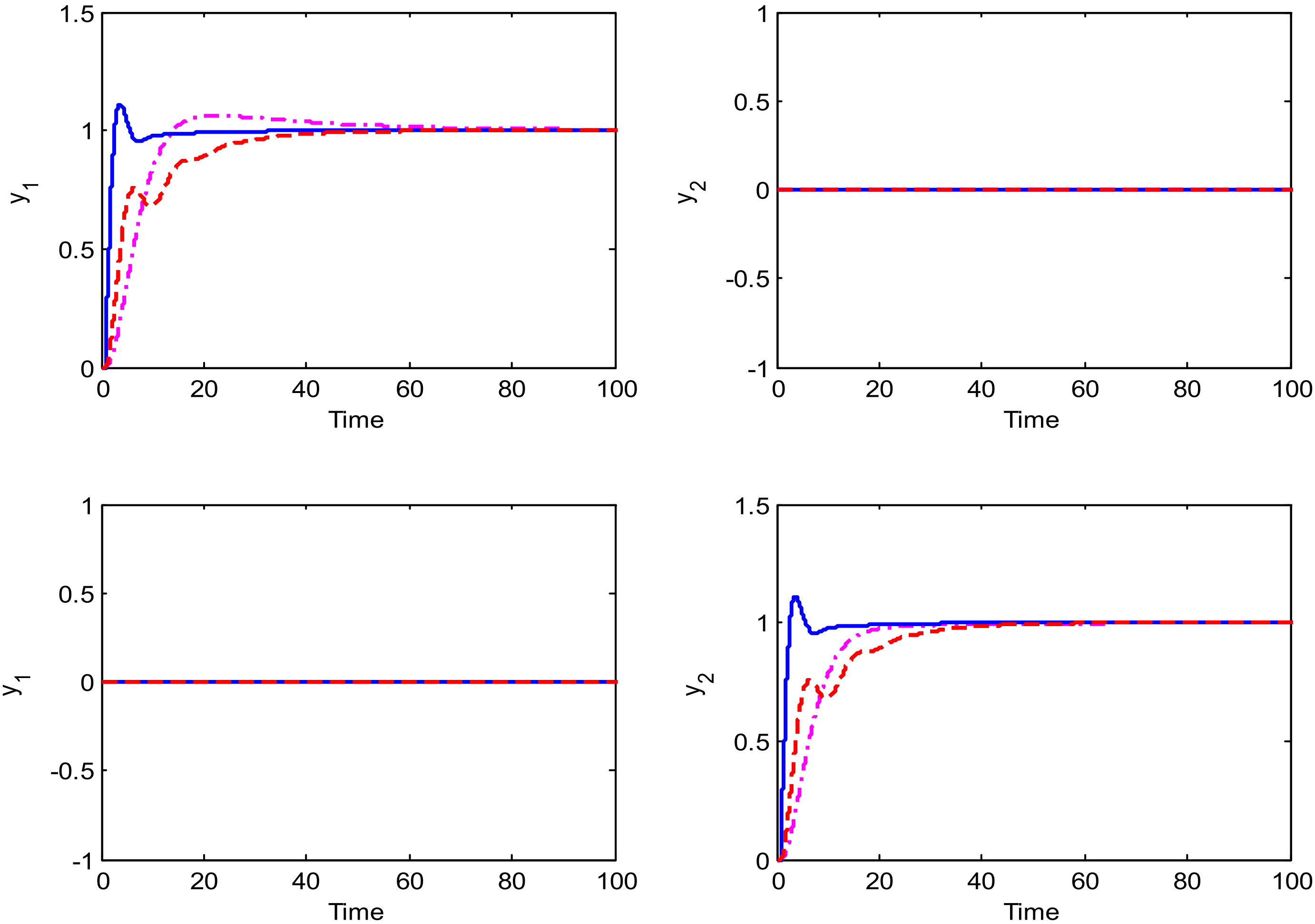

Fig. 5. Comparison of servo responses for a unit step input in y_1 and y_2, proposed method (solid), Hazarika (dash) and Dasari et al. (dash dot) for example 2.

Table 6
Comparison of IAE values of servo responses with various perturbations in parameters among proposed method, Hazarika method and Dasari et al. method for example 2.

Perturbations	Proposed method				Hazarika method				Dasari et al. method			
	Unit step input in loop1		Unit step input in loop2		Unit step input in loop1		Unit step input in loop2		Unit step input in loop1		Unit step input in loop2	
	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2
k_p	2.18	0	0	2.17	7.91	0	0	7.89	8.8	0.01	0	7.734
$1.1k_p$	4	0.01	0	3.98	10.43	0.01	0	10.42	9.22	0.01	0	9.30
$0.9k_p$	4.13	0.01	0	4.15	4.82	0.01	0	4.8	9.87	0.011	0	7.39
τ	2.18	0	0	2.17	7.91	0	0	7.89	8.8	0.01	0	7.734
1.1τ	1.91	0.01	0	1.91	7.91	0.01	0	7.89	9.31	0.01	0	7.74
0.9τ	2.66	0.01	0	2.65	7.91	0.01	0	7.89	8.35	0.01	0	7.74
θ	2.18	0	0	2.17	7.91	0	0	7.89	8.8	0.01	0	7.734
1.1θ	2.9	0.01	0	2.89	7.91	0.01	0	7.89	8.47	0.01	0	7.74
0.9θ	1.77	0.01	0	1.76	7.91	0.01	0	7.89	9.12	0.01	0	7.74

7. Conclusions

In the present work, improved controller designs with simple tuning rules for TITO unstable processes using the previously developed decoupled control scheme are proposed. The performance of the designed controllers is analyzed with different theoretical examples. Performance of the designed controller is much better than that of the previously existing methods. Even though interactions are slightly more in some cases for the proposed design, on a whole, the proposed designed controller acts better than both Hazarika and Chidambaram [12] method and Dasari et al [11] method. Proposed control system is showing improved responses in the case of set point tracking and disturbance rejection. The proposed method consists of

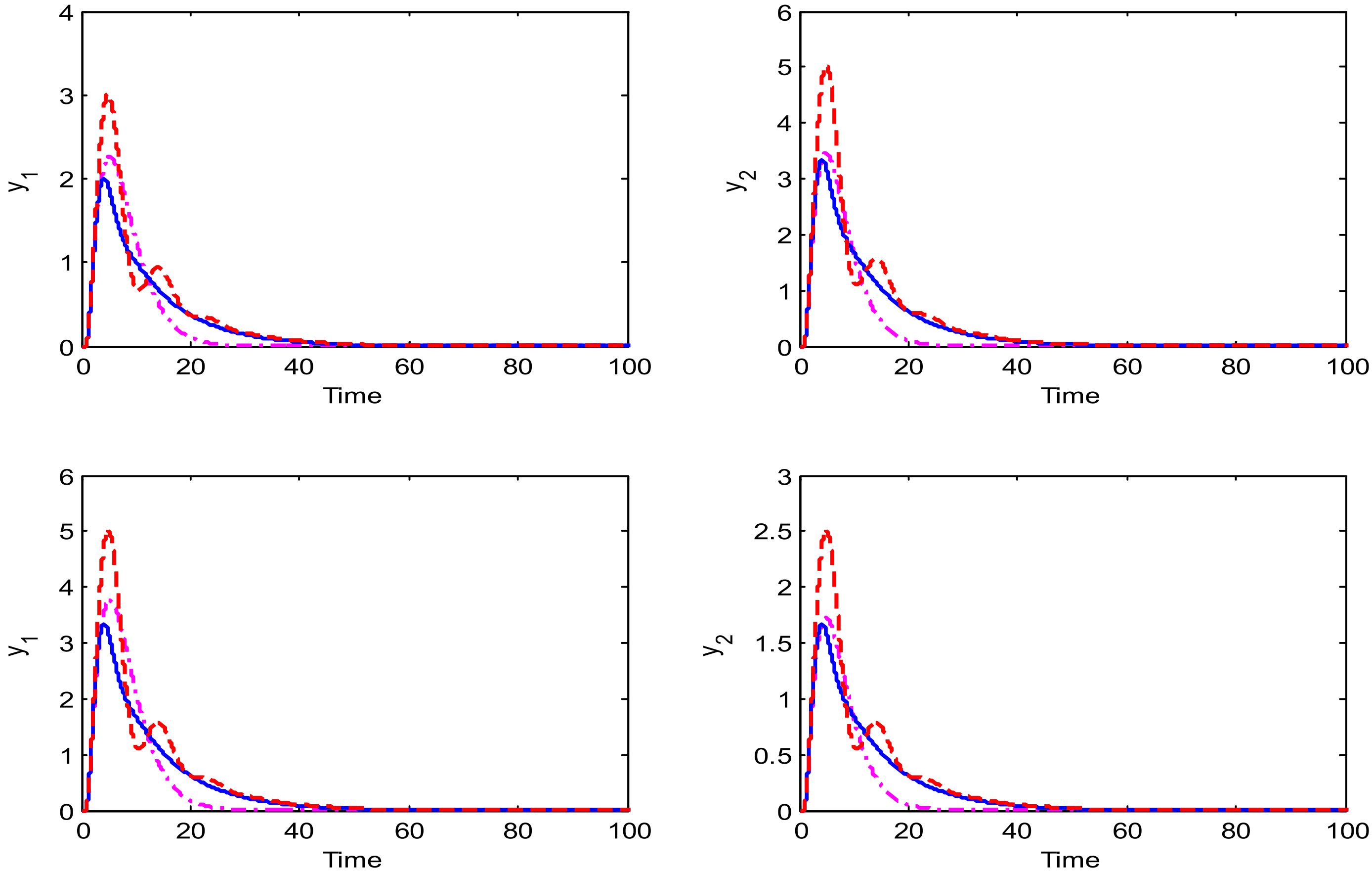

Fig. 6. Comparison of regulatory responses for a unit step input in y1 and y2, proposed method (solid), Hazarika (dash) and Dasari et al. (dash dot) for example 2.

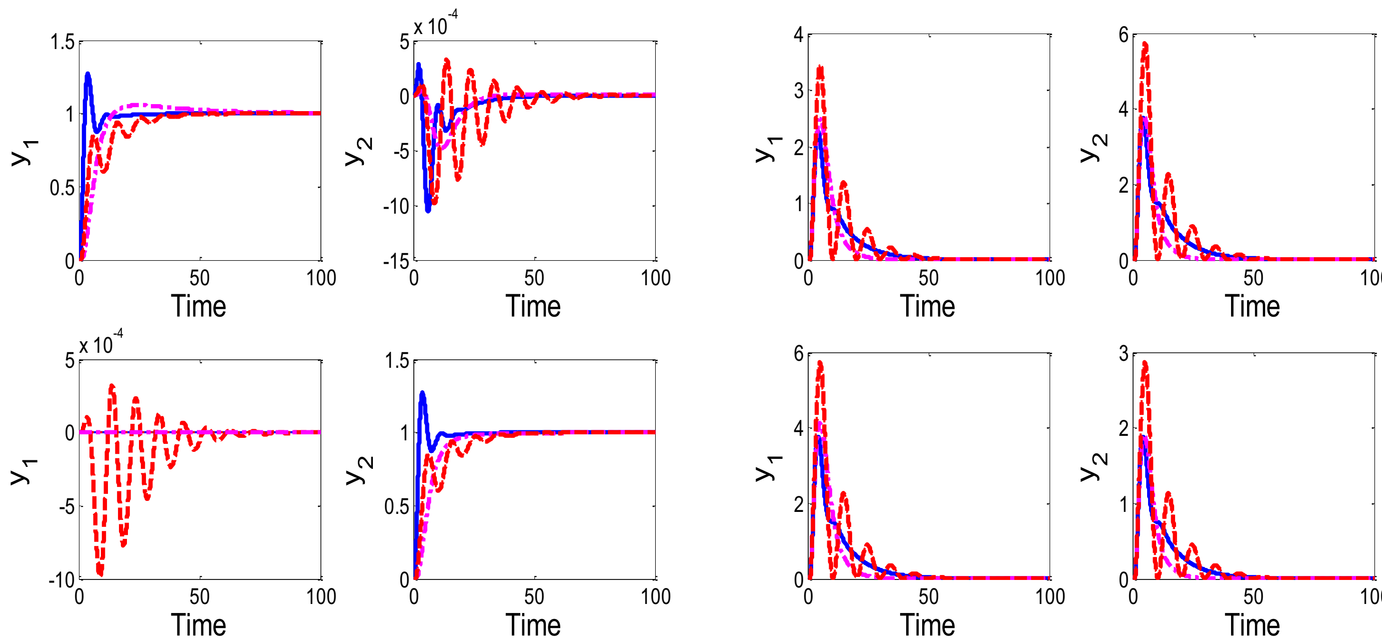

Fig. 7. Comparison of servo and regulatory responses with 10% perturbations to time delay for a unit step input in y_1 and y_2, proposed method (solid), Hazarika et al. (dash) and Dasari et al. (dash dot) for example 2.

Table 7
Comparison of IAE values of regulatory responses with various perturbations in parameters among proposed method, Hazarika method and Dasari et al. method for example 2.

Perturbations	Proposed method				Hazarika method				Dasari et al. method			
	Unit step input in loop1		Unit step input in loop2		Unit step input in loop1		Unit step input in loop2		Unit step input in loop1		Unit step input in loop2	
	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2
k_p	22.66	37.78	37.76	18.88	27.81	46.37	46.35	23.17	20.58	28.75	34.29	14.37
$1.1k_p$	22.65	37.77	37.74	18.87	27.79	46.34	46.32	23.16	20.58	28.75	34.29	14.37
$0.9k_p$	22.66	37.78	37.77	18.88	27.81	46.37	46.35	23.17	27.44	37.05	45.73	18.47
τ	22.66	37.78	37.76	18.88	27.81	46.37	46.35	23.17	20.58	28.75	34.29	14.37
1.1τ	22.66	37.78	37.76	18.88	27.81	46.37	46.35	23.17	20.86	29.15	34.77	14.56
0.9τ	22.66	37.78	37.76	18.88	27.81	46.37	46.34	23.17	20.58	28.75	34.29	14.37
θ	22.66	37.78	37.76	18.88	27.81	46.37	46.35	23.17	20.58	28.75	34.29	14.37
1.1θ	22.66	37.78	37.76	18.88	27.81	46.37	46.34	23.17	20.58	28.75	34.29	14.37
0.9θ	22.66	37.78	37.76	18.88	27.81	46.37	46.35	23.17	20.72	28.96	34.54	14.47

only one controller for each loop whereas in the previous method such as Hazarika and Chidambaram [12], at least two controllers were used in each loop. The ability to provide good stable closed loop response even when there are large amount of perturbations in the process parameters is a major advantage of the proposed method over previously existing methods. Quantitative comparison is carried out using IAE values and the proposed method is superior over Hazarika and Chidambaram [12] method and Dasari et al. [11] method.

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