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Research paper

A comparative study of fractional order $PI^{\lambda}/PI^{\lambda}D^{\mu}$ tuning rules for stable first order plus time delay processes

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Abstract

Conventional PID tuning methods may not be sufficient to deal with complex processes of modern industry. For better control, fractional order $PI^{\lambda}D^{\mu}$ controller was introduced as the generalization of classical PID controller with the help of non-integer order (fractional order) calculus. The fractional calculus uses integration and differentiation with a fractional order or complex order. The major advantage of fractional derivative is the ability to inherit the nature of the processes. In general, the control loop includes both fractional order process model and fractional order controller. However, the processes to be controlled are usually modeled as integer order models and controlled using fractional order controllers. But if the plant model is obtained as fractional model, it is converted into integer order model by approximating the fractional terms using different approximations proposed in the literature. With all the above mentioned advantages, several fractional order $PI^{\lambda}/PI^{\lambda}D^{\mu}$ tuning rules are proposed in the literature for integer order systems and researchers are still proposing the new rules. The main aim of this paper is to compare fractional order PI/PID tuning methods based on Integral of Absolute Error (IAE), Total Variation (TV) and Maximum Sensitivity (M_s). The main reason for choosing fractional order $PI^{\lambda}/PI^{\lambda}D^{\mu}$ controllers is their additional degrees of freedom that result in better control performance. These tuning rules were applied on several first order plus time delay processes subjected to step change in setpoint and disturbance.

Six recent tuning methods, three for fractional order PI^{λ} and the remaining for fractional order $PI^{\lambda}D^{\mu}$, were considered. Finally, from the simulation results the optimal tuning method is recommended based on the control objective of the process and the process dead time (L) to time constant (T) ratio. It is observed that the performance of tuning methods vary with the nature of the process like lag dominant, balanced and delay significant processes. The FOPTD processes were checked for robustness with increasing L/T ratio with respect to IAE, TV and M_s.

Keywords: Fractional order; Tuning; Robustness; Sensitivity; Integral of absolute error; Maximum sensitivity

1. Introduction

The past six decades have seen the development of algorithms for tuning PID controllers and their application in industry. The reason for the success of PID controllers is its lucid transfer function, the availability of good number of effective tuning rules and their acceptance by the industrial world based on minimum knowledge of the process to be controlled. The most widely used controller in the industry is PID due to its simple structure and auto tuning capability. Although there are many rule based methods and analytical tuning methods, it is difficult to adjust PID parameters properly to meet the requirements because many industrial systems are often burdened with problems such as structural complexity, uncertainties, large transportation lags and nonlinearities. The closed loop performance of such systems can be enhanced using $PI^{\lambda}D^{\mu}$ controller, providing additional degrees of freedom with fractional order of the integral λ and derivative μ. This flexibility allows designing of more robust systems, thus providing the precise control of real world processes. The strength of FOPID controller can be attributed to its robustness, i.e., less sensitive to uncertainties in the process parameters and the ability to control both integer order and fractional order dynamical systems. In the field of automatic control, these advantages would lead to more precise and robust control of dynamical systems.

Better control is always a primary objective in the field of control engineering and opens new boundaries for developing

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much better control algorithms. The fractional order proportional integral derivative (FOPID or $PI^{\lambda}D^{\mu}$) controller has been a topic of interest of researchers from academia and industry in the last ten years. They are more flexible and provide improved performance than conventional PID controllers due to the presence of five parameters to be tuned (rather than three in the case of traditional PID). However, the tuning of FOPID controller becomes complex. To address these problems, different design and tuning algorithms have been proposed in the literature for PI^{λ}/ $PI^{\lambda}D^{\mu}$ controllers.

A closed loop control system encounters different combinations of plant and controllers while handling real world problems. They include the integer or fractional order of either plant or controller or both. In practice, the plant models have been obtained as integer order models and it is natural to consider the fractional nature of the controller. The efforts of control engineers and scientists lead to the development of fractional order controller ($PI^{\lambda}/PI^{\lambda}D^{\mu}$) tuning rules. Podlubny [1] had proposed the fractional order PI and PID controllers that demonstrated better response with integrator and differentiator rose to the fractional powers λ and μ respectively. There are several other PI^{λ} and $PI^{\lambda}D^{\mu}$ controllers in the literature [2–6] whose tuning rules are developed using evolutionary algorithms by minimizing objective functions. Some heuristics based controllers are also reported.

Many industrial processes are better approximated as first order plus time delay (FOPTD) models. In fact, there are rules to approximate higher order processes and processes with nonlinearity to fit to FOPTD models [7]. This paper is restricted to the use of integer order FOPTD models due to the above mentioned reasons for controller design. This study determines the optimal $PI^{\lambda}/PI^{\lambda}D^{\mu}$ tuning rules to calculate controller settings for stable FOPTD processes. The work also suggests the suitable tuning method depending on the control objective. The closed loop performance of two lag dominant FOPTD processes is studied. Further, the variation of performance indices and robustness of the controller are considered.

In the current work, the following six $PI^{\lambda}/PI^{\lambda}D^{\mu}$ tuning rules were considered based on the review of recent tuning rules in the literature. Initially, the FOPI tuning rules were studied [8–10]. The FOPID tuning rules [11–13] were then chosen for study as they reduce the effect of overshoot caused by integral action in FOPI. The present work compares the tuning rules with regard to performance metrics integral of absolute error (IAE), total variation (TV) and maximum sensitivity (M_s) for integer order FOPTD models.

2. Background

2.1. Block diagram of closed loop control system

The general feedback loop used for the control of single input single output processes is shown in Fig. 1. The various parameters in the loop are: y_{sp} – set value, y – controlled variable, u – manipulated variable and $e(=y_{sp}-y)$ – the error. Another input variable which enters the process and affects the controlled variable is the disturbance ‘d’. $G_p(s)$ represents a stable process which includes the dynamics of measuring element and final control element and is modeled by an FOPTD transfer function:

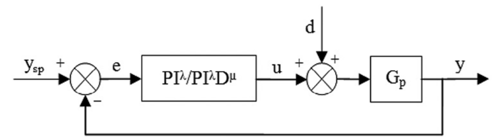

Fig. 1. Feedback control loop with $PI^{\lambda}/PI^{\lambda}D^{\mu}$ controller.

$$G_p(s) = \frac{K}{Ts+1}e^{-Ls} \tag{1}$$

The $PI^{\lambda}/PI^{\lambda}D^{\mu}$ block contains the transfer function of controller and is represented in ideal form or parallel form as follows:

$$\begin{aligned} G_c(s) &= K_p\left(1+\frac{1}{T_i s^{\lambda}}\right) = K_p + \frac{K_i}{s^{\lambda}} \text{ or} \\ G_c(s) &= K_p\left(1+\frac{1}{T_i s^{\lambda}} + T_d s^{\mu}\right) = K_p + \frac{K_i}{s^{\lambda}} + K_d s^{\mu} \end{aligned} \tag{2}$$

The series form of $PI^{\lambda}D^{\mu}$ controller considered in the literature is:

$$K_p\left(\frac{T_i s^{\lambda}+1}{T_i s^{\lambda}}\right)\left(\frac{T_d s^{\mu}+1}{(T_d/N)s+1}\right); N = 10T^{\mu-1} \tag{3}$$

Chen et al., Gude et al., and Bhambhani et al. used the parallel form of FOPI controller [8–10]. Also, Valerio et al., and Bayat used the parallel form of FOPID controller, whereas Padula et al. used the series form of FOPID controller [11–13]. Series form and parallel forms of controller structures are used according to the application. It is observed that the parallel form of PID controller is more often used in academia and in research while the series form of PID controller is used in industrial controllers. The ultimate purpose of both forms of controllers is the same and it is possible to convert controller structure from one form to the other using associated relations.

2.2. Performance assessment of the controller

The generally used measures for controller performance are integral error criteria: integral of squared error (ISE), integral of absolute error (IAE) and integral of time weighted absolute error (ITAE). In the current study, IAE criterion was chosen as performance metric for the controller because the minimization of IAE results in small overshoot and low settling time in the closed loop response of the system when there is a step change in set point or load disturbance. It is defined as:

$$IAE = \int_0^{\infty} |e(t)|\,dt \tag{4}$$

Another controller performance metric is the total variation (TV) of the manipulated variable. Total variation is an indication of the smoothness of the manipulated variable. Total variation is defined as:

$$TV=\sum_{i=0}^{\infty}\left|u_{i+1}-u_i\right| \tag{5}$$

For effective and stable function of the closed loop control system, it should not be sensitive to process parameter changes or model uncertainties. This is a major requirement for robustness of the controller. The sensitivity function S is defined as:

$$S(j\omega)=\frac{1}{1+G_c(j\omega)G_p(j\omega)} \tag{6}$$

This sensitivity function tells us how a closed loop system is influenced by parameter variations in the process. The maximum value of the sensitivity function is a performance metric for robustness analysis:

$$M_s=\max_{0<\omega<\infty}\left|S(j\omega)\right| \tag{7}$$

The value of maximum sensitivity M_s can also be found from the Nyquist stability test of closed-loop system. For the stability of closed loop system, the Nyquist curve should not enclose the point (−1, j0). The maximum sensitivity is the reciprocal of shortest distance between Nyquist curve and the point (−1, j0). If the value of M_s increases, the system becomes less robust to modeling uncertainties. The typical range of maximum sensitivity should be 1 to 2 to ensure robust performance of the controller.

3. $PI^{\lambda}/PI^{\lambda}D^{\mu}$ tuning rules

This section describes six tuning methods considered in this work and the formulas for controller settings of all the tuning methods are listed in Tables 1 and 2.

3.1. Chen et al. [8] method

This method proposed tuning parameters of the controller by optimizing the regulatory response with a constraint on the maximum load disturbance-to-output sensitivity M_s.

3.2. Gude et al. [9] method

The tuning rules are developed by minimizing a performance criteria J_v in frequency domain with a constraint on the maximum sensitivity. J_v measures the ability of the closed loop system to handle disturbance inputs of low frequencies.

$$J_v=\left\|\frac{1}{s}G(s)S(s)\right\|_\infty=\max_\omega\left|\frac{1}{j\omega}\frac{G(j\omega)}{1+L(j\omega)}\right| \tag{8}$$

3.3. Bhambhani et al. [10] method

These tuning rules were proposed based on the minimization of two controller performance indices: Jitter margin and ITAE. Multi-objective optimization algorithm is used to calculate the optimum values of the tuning parameters.

$$\text{Jitter margin}=\frac{1}{\delta_{max}};\ \delta_{max}=\min_{0<\omega<\infty}\left|\frac{1+G(j\omega)C(j\omega)}{j\omega G(j\omega)C(j\omega)}\right| \tag{9}$$

$$ITAE=\int_0^\infty t\left|e(t)\right|dt \tag{10}$$

3.4. Valerio et al. [11] method

These rules are applicable for processes whose unit step response is of s-shaped. The advantage of this method is that the controller design does not require the plant data. Two sets of tuning rules were proposed: first set of tuning rules are applicable for processes with $0.1\le T\le 50$and $L\le 2$, and second set for $0.1\le T\le 50$ and $L\le 0.5$. A numerical optimization algorithm namely Nelder–Mead's simplex method is implemented in MATLAB to minimize the condition:

$$\left|G(j\omega_{cg})C(j\omega_{cg})\right|=0\text{dB} \tag{11}$$

These rules are easier to apply but perform worse than analytical tuning methods. Fine tuning is often required for controllers designed based on these tuning rules.

Table 1
Fractional order PI controller tuning rules.

Tuning rule	Controller settings				
Chen et al. [8]	$K_p=\frac{1}{K}\left(\frac{0.2978}{\tau+0.000307}\right)$	$T_i=T\left(\frac{0.8578}{\tau^2-3.402\tau+2.405}\right)$			$\lambda=\begin{cases}1.1 & \text{if } \tau\ge 0.6\\ 1.0 & \text{if } 0.4\le\tau<0.6\\ 0.9 & \text{if } 0.1\le\tau<0.4\\ 0.7 & \text{if } \tau<0.1\end{cases}$
Gude et al. [9]	$f(\tau)=a\tau^b+c$	a	b	c	τ
	KK_p	0.2154	−1.169	−0.1592	0 < τ < 1
	aK	−0.4645	0.3182	0.5795	0 < τ < 0.25
		3.271	5.75	0.28	0.25 < τ < 1
	T_i/L	9.242	−0.1966	−9.171	0 < τ 1
	T_i/T	5.479	0.8154	−0.03853	0 < τ < 0.3
		6.06	7.066	1.18	0.3 < τ < 1
	λ	1.12			0 < τ < 1
Bhambhani et al. [10]	$K_p=\frac{0.2T}{L}+0.16$	$K_i=\frac{0.25}{TL}+\frac{0.19833}{L}+0.09$			$\lambda=\tau-0.04L+1.2399$

Table 2
Fractional order PID controller tuning rules.

Valerio et al. [11]

First set of tuning rules

	For $0.1 \le T \le 5$ and $L \le 2$					For $5 \le T \le 50$ and $L \le 2$				
	P	I	λ	D	μ	P	I	λ	D	μ
1	−0.0048	0.3254	1.5766	0.0662	0.8736	2.1187	−0.5201	1.0645	1.1421	1.29
L	0.2664	0.2478	−0.2098	−0.2528	0.2746	−3.5207	2.6643	−0.3268	−1.3707	−0.5371
T	0.4982	0.1429	−0.1313	0.1081	0.1489	−0.1563	0.3453	−0.0229	0.0357	−0.0381
L^2	0.0232	−0.133	0.0713	0.0702	−0.1557	1.5827	−1.0944	0.2018	05552	0.2208
T^2	−0.072	0.0258	0.0016	0.0328	−0.025	0.0025	0.0002	0.0003	−0.0002	0.0007
LT	−0.0348	−0.0171	0.0114	0.2202	−0.0323	0.1824	−0.1054	0.0028	0.263	−0.0014

Second set of tuning rules
For $0.1 \le T \le 50$ and $L \le 0.5$

	P	I	λ	D	μ
1	−1.0574	0.6014	1.1851	0.8793	0.2778
L	24.542	0.4025	−0.3464	−15.0846	−2.1522
T	0.3544	0.7921	−0.0492	−0.0771	0.0675
L^2	−46.7325	−0.4508	1.7317	28.0388	2.4387
T^2	−0.0021	0.0018	0.0006	−0.0000	−0.0013
LT	−0.3106	−1.205	0.038	1.6711	0.0021

Bayat [12]

	Servo control	Regulatory control
K_p	$\frac{1}{K}\left[\frac{0.3663\tau+0.8856}{\tau+0.000792}\right]$	$\frac{1}{K}\left[\frac{0.2709\tau+0.566}{\tau-0.0364}\right]$
T_i	$T\,(0.3827\tau + 0.9354)$	$T\left(1.252\tau^{0.5555}-0.05696\right)$
T_d	$T\left(0.5036\tau^{0.7152}-0.07974\right)$	$T\,(0.3425\tau + 0.02753)$
λ	1	1
μ	$1.095-0.03625\tau$	$2.503-1.368\tau^{0.04705}$

Padula et al. [13]
For $M_s = 1.4$

For servo control				For regulatory control			
$f(\tau)$	a	b	c	a	b	c	$K_p = \frac{1}{K}\left(a\tau^b + c\right)$
K_p	0.6503	−0.9166	−0.6741	0.2776	−1.095	−0.1426	$T_i = T^{\lambda}\left(a\left(\frac{L}{T}\right)^b + c\right)$
T_i	0.04701	−0.2611	0.9276	0.6241	0.5573	0.0442	$T_d = T^{\mu}\left(a\left(\frac{L}{T}\right)^b + c\right)$
T_d	0.3563	1.2	0.0003108	0.4793	0.7469	−0.02393	
λ	1			1			
μ	1.1 for $\tau < 0.1$			1.0 for $\tau < 0.1$			
	1.2 for $\tau \ge 0.1$			1.1 for $0.1 \le \tau < 0.4$			
				1.2 for $\tau \ge 0.4$			

3.5. Bayat [12] method

These tuning rules were proposed based on the dimensional analysis and are applicable for processes modeled as FOPTD. The only limitation is that the normalized dead time should be between 0.1 and 3.5. Tuning rules for command following and disturbance rejection were proposed exclusively to minimize ISE performance index.

3.6. Padula et al. method

Padula et al. [13] proposed FOPID tuning rules explicitly for servo control and regulatory control of closed loop system by minimizing integrated absolute error (IAE) with a constraint on the maximum sensitivity M_s. The series form of FOPID controller was considered for proposing the tuning rules. The tuning rules were developed by considering several processes with various dead time values. The values of the tuning parameters had been found by using genetic algorithm. Finally, the optimal tuning rules were obtained by interpolating the optimal values found for controller parameters of different processes with different dead time τ. The structure of the tuning parameters devised is given as follows:

$$K_p = \frac{1}{K}\left(a\tau^b + c\right) \tag{12}$$

$$T_i = T^{\lambda}\left(a\left(\frac{L}{T}\right)^b + c\right) \tag{13}$$

$$T_d = T^{\mu}\left(a\left(\frac{L}{T}\right)^b + c\right) \tag{14}$$

4. Simulation and comparison study

A stable FOPTD model is considered for the current study [14]:

$$G_p(s) = \frac{1}{s+1}e^{-Ls} \tag{15}$$

For analogy, the performance of controller tuning methods is compared for variations in load and set point when they undergo a step change of unit magnitude. L/T ratio is a significant factor which affects the controller performance and sensitivity of the feedback control system. The effect of L/T ratio on different tuning methods was studied by varying time delay L so that the ratio L/T varies from 0.1 to 2 covering lag dominant, balanced and delay significant processes. The simulations were carried out on different FOPTD processes. The main reason for varying the L/T ratio is that it affects the robustness of controller and performance of the closed loop system. For each variation of L/T, new controller settings are calculated and closed loop response (both servo and regulatory) is observed, thus recording IAE, TV and M_s. The trends of the performance indices IAE, TV and M_s are observed for the entire range of L/T ratio. Finally, the behavior of IAE, TV and M_s is observed after plotting them against L/T ratio. Also, the performance of the fractional controllers was observed by adjusting the fractional tuning parameters λ and μ. All the simulations are carried out using MATLAB and Simulink.

4.1. Case study: liquid level system

To verify the recommended tuning rules, the case study of liquid level system modeled as FOPTD [15] is considered. The transfer function of liquid level system modeled as FOPTD system is:

$$G_p(s) = \frac{3.13}{433.33s+1}e^{-50s} \tag{16}$$

The value of $L/T = 0.1034 < 1$ indicates that the system is a lag dominant process and the behavior of performance indices are observed for the variation of L/T ratio.

5. Results and discussion

The simulation studies on servo control and regulatory control problems are presented for two stable FOPTD processes. The IAE, TV and M_s values are expected to be minimum for better performance of the controller. The servo response of the closed loop system and the corresponding FOPI and FOPID controller responses are shown in Fig. 2 for lag dominant process. It is evident from the response that the Gude et al. [9] method provides better response. The Chen et al. [8] method also provides a good response but left with an offset in the response. This is due to the low value of fractional order of the integral action, i.e., 0.9 as the normalized dead time is 0.2307. The Bhambhani et al. [10] method is giving an oscillatory response. It is clear from the response of the closed loop system with FOPID controller that the Padula et al. [13] method results in a stable and offset free response without any overshoot while the other two methods produce an oscillatory response. There appears an offset in the response when the controller is designed with Valerio et al. [11] tuning rules because the fractional order λ of the integral is 0.8281. Further, the performance of the controllers was observed by varying the fractional parameters λ and μ over a possible range values, starting with a value that gives a stable response (Fig. 3).

5.1. Servo control

5.1.1. Servo control of lag dominant process

The stable FOPTD model considered for the study becomes lag dominant for any value of time delay, between 0.1 and 0.9.

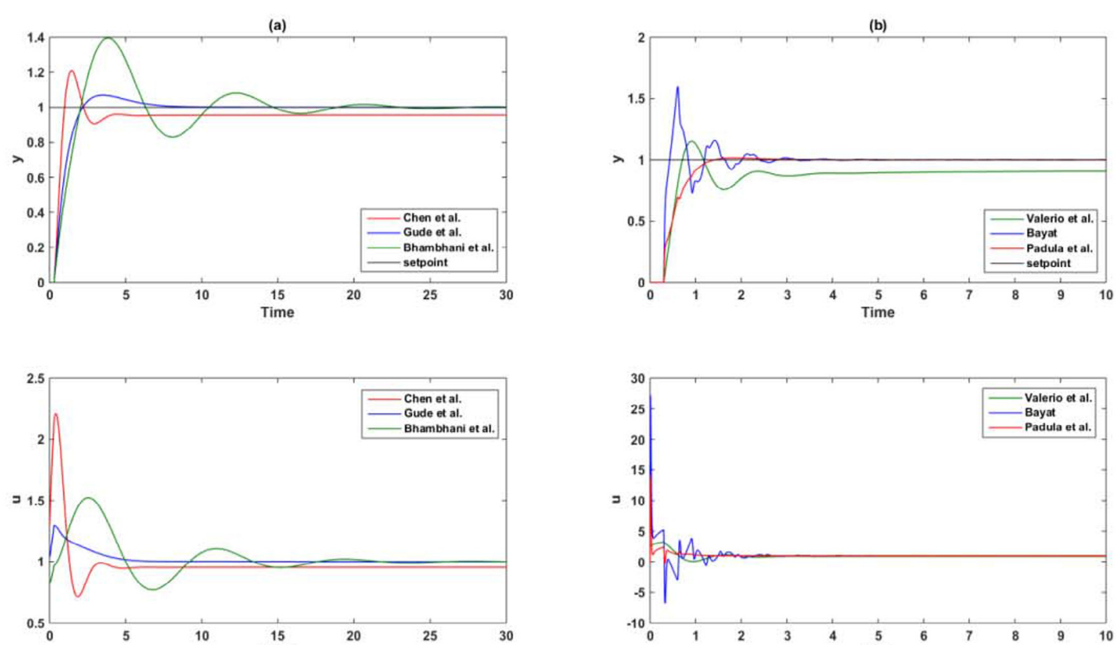

Fig. 2. Closed loop responses of lag dominant process for set-point change and L/T = 0.3 (a) FOPI, (b) FOPID.

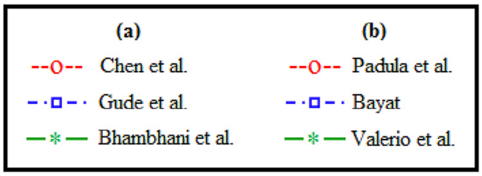

Fig. 3. (a) Legend for Figs 4–6,10–12,16–17 and 20–21. (b) Legend for Figs 7–9,13–15,18–19 and 22–23.

Here, it is chosen as 0.3 so that the ratio L/T becomes 0.3. Fig. 4 shows the variation of performance metrics IAE and TV of the FOPI controller as L/T varied from 0.1 to 2. Fig. 5 shows the robustness M_s of the controller for servo control problem. The Chen et al. [8] method gives lower values of IAE and TV whereas the Gude et al. [9] method is robust to model uncertainties compared to the other two methods. Both the methods are robust to model parameter uncertainties because the maximum sensitivity lies in the range of 1.2 to 2. The controller effort in terms of total variation is the same in case of both methods. The Bhambhani et al. [10] method fails to be robust to model uncertainties as it gives much higher values of M_s compared to the other two methods and also the higher values of IAE. Fig. 5 also clarifies that the Gude et al. [9] method performs better compared to the other two methods even if the tuning parameter λ is varied over a range of values keeping others unaltered. The Chen et al. [8] method can be a second choice for good system performance if offset is tolerated (Fig. 6).

The performance of FOPID controller is presented in Figs 7–9 for variation of controllability index L/T. Lower values of IAE and TV are observed with the Padula et al. [13] method. The Valerio et al. [11] method shows significantly higher values compared to Padula et al. [13] method. The Bayat [12] method shows poor performance both in terms of IAE and TV. The robustness of the controller presented in Fig. 8 indicates that the Padula et al. [13] method is robust to model uncertainties. FOPID controller performance presented in Fig. 9 with the variation of tuning parameter λ confirms that the Padula et al. [13] method is superior in terms of IAE, TV and M_s. The Valerio et al. [11] method can be a second choice but its application is limited to the systems with s-shaped step response; time delay must be less than or equal to 2 and time constant should be less than or equal to 50.

5.2. Regulatory control

5.2.1. Regulatory control of lag dominant process

Figures 10 and 11 show the variation of performance metrics IAE, TV of the FOPI controller and sensitivity for regulatory control problem. It is observed that the Gude et al. [9] method results in lower values of TV whereas the Chen et al. [8] method gives lower values of IAE but with an offset in the response as long as the fractional order of the integrator is below 1. The graphs illustrated in Fig. 11 reveal that the Gude et al. [9] method is insensitive to the model parameter variations and the Chen et al. [8] method too as its M_s values are also in the range of 1.2 to 2. Finally, the Gude et al. [9] method is recommended

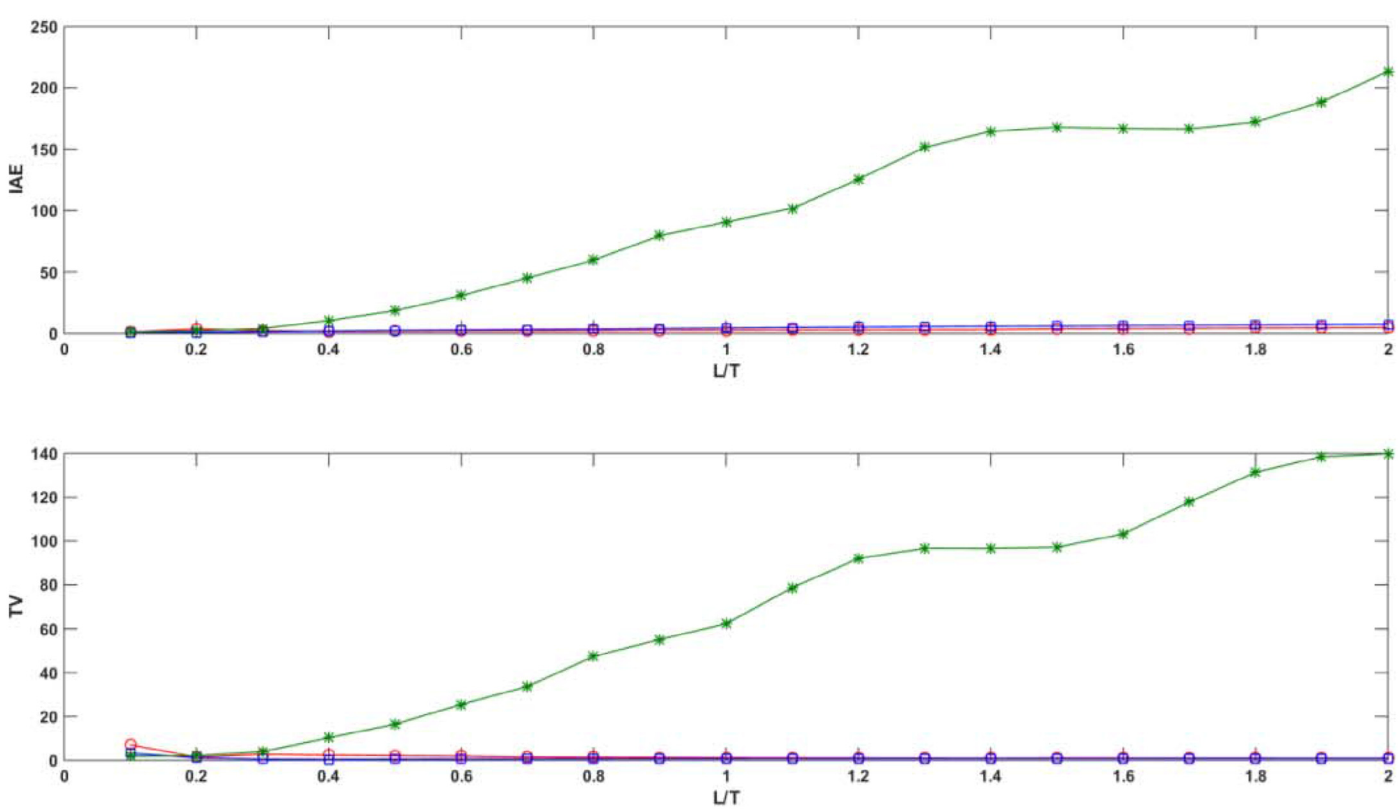

Fig. 4. Increasing L/T ratio versus performance metrics of FOPI controller for step set point changes of unit magnitude for the three tuning methods.

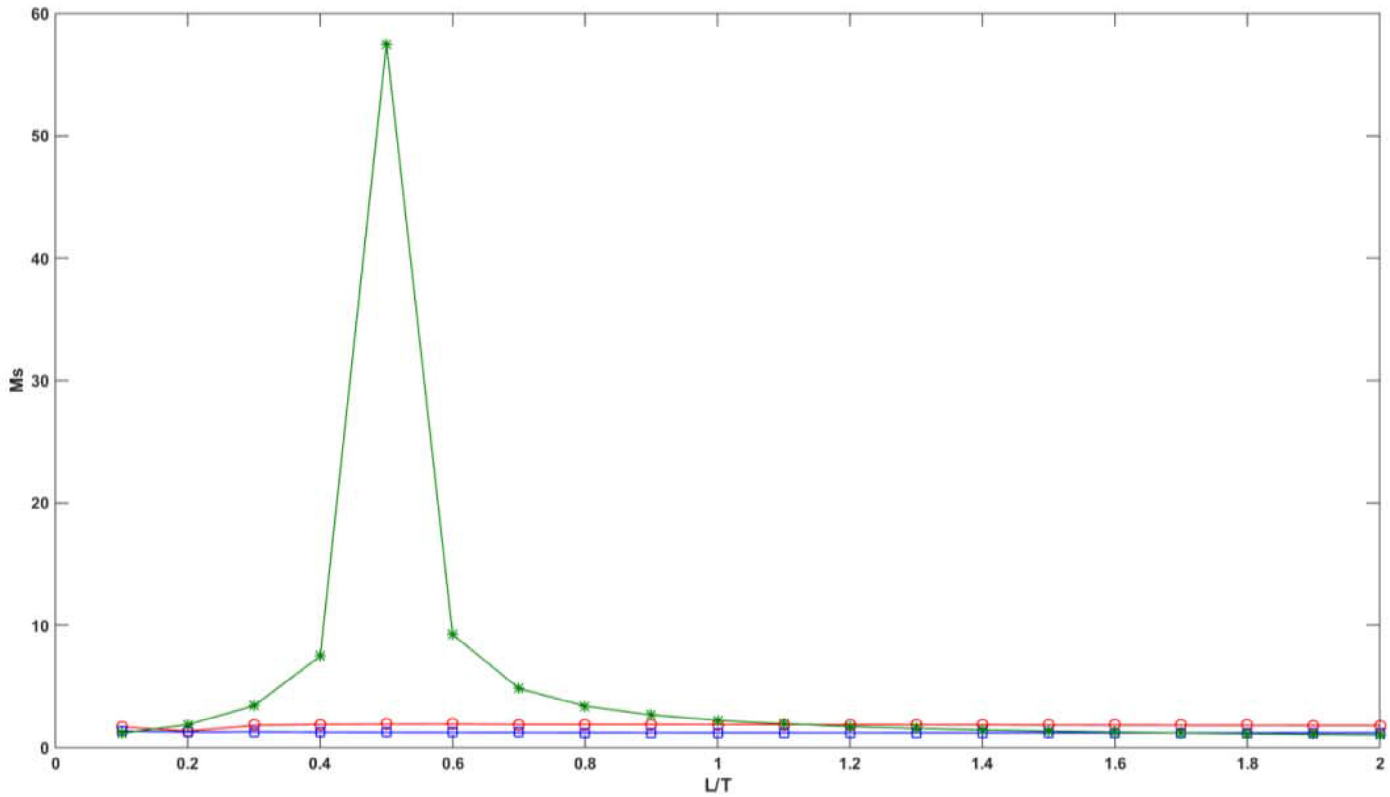

Fig. 5. Robustness of FOPI controller for step set point changes of unit magnitude with increasing L/T ratio for the three tuning methods.

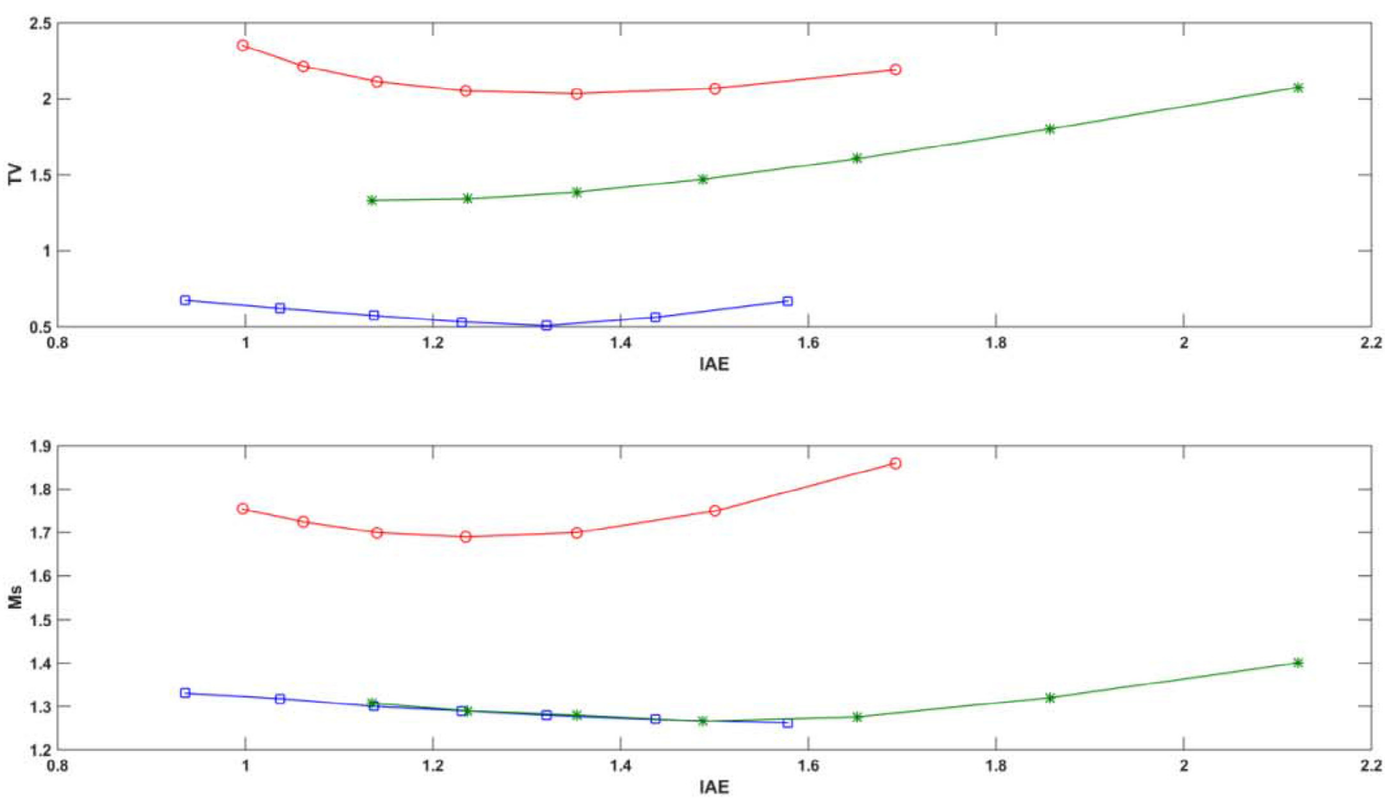

Fig. 6. FOPI controller performance for step set point changes of unit magnitude with L/T ratio of 0.3 for the three tuning methods.

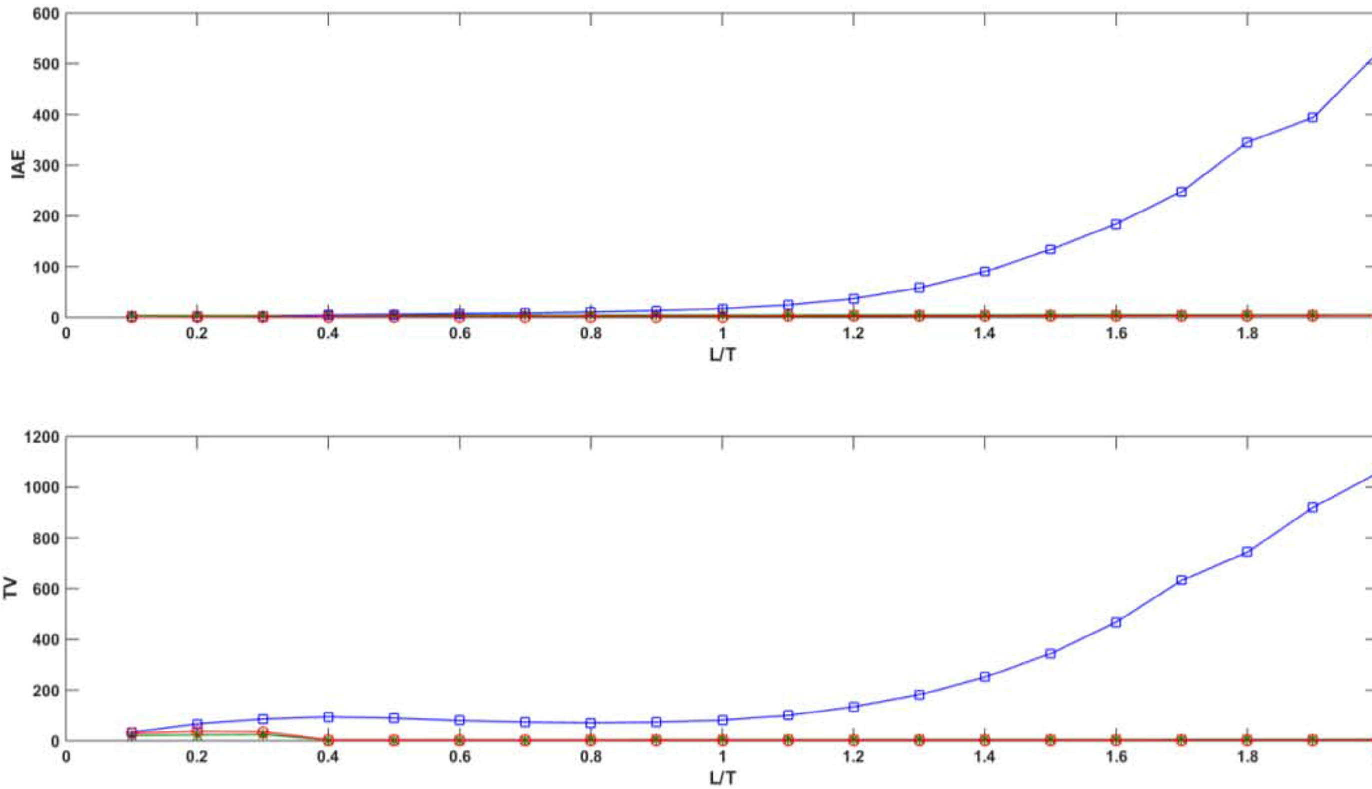

Fig. 7. Increasing L/T ratio versus performance metrics of FOPID controller for step set point changes of unit magnitude for the three tuning methods.

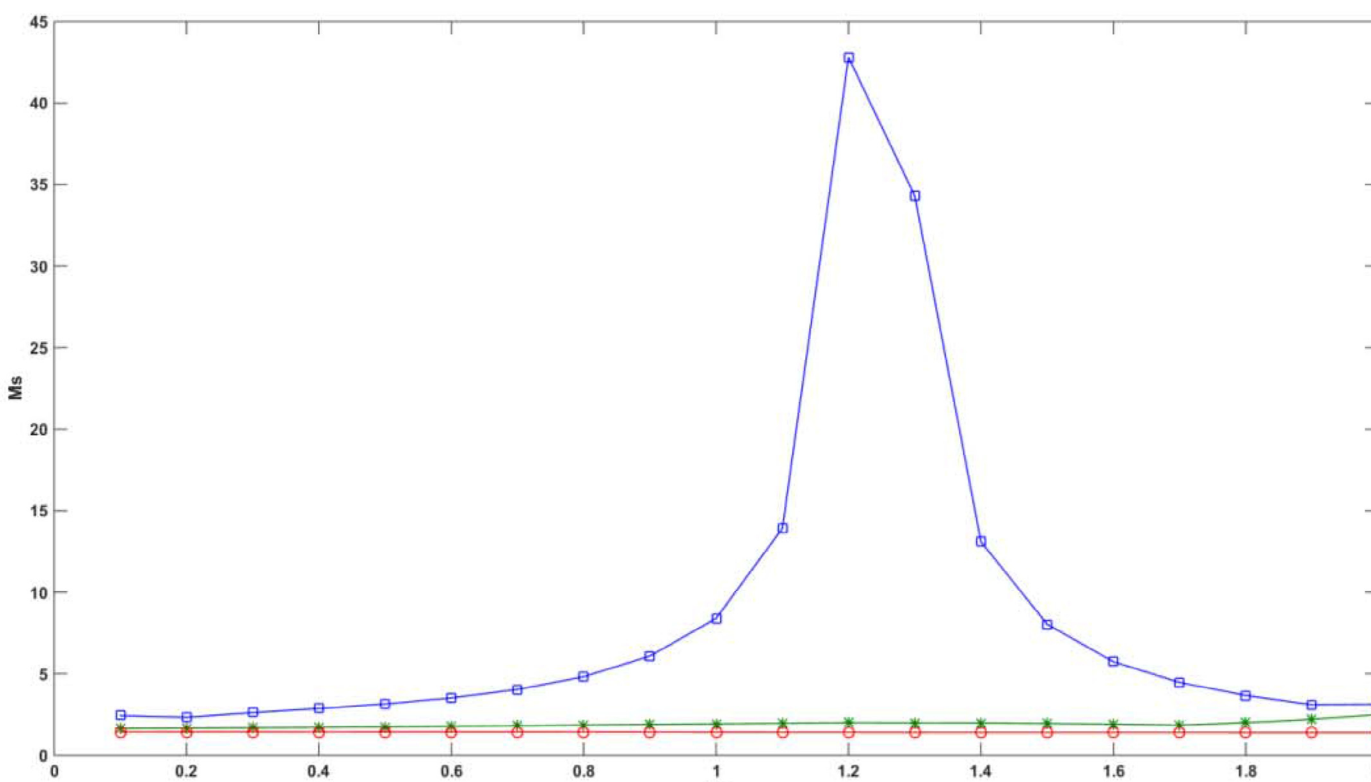

Fig. 8. Robustness of FOPID controller for step set point changes of unit magnitude with increasing L/T ratio for the three tuning methods.

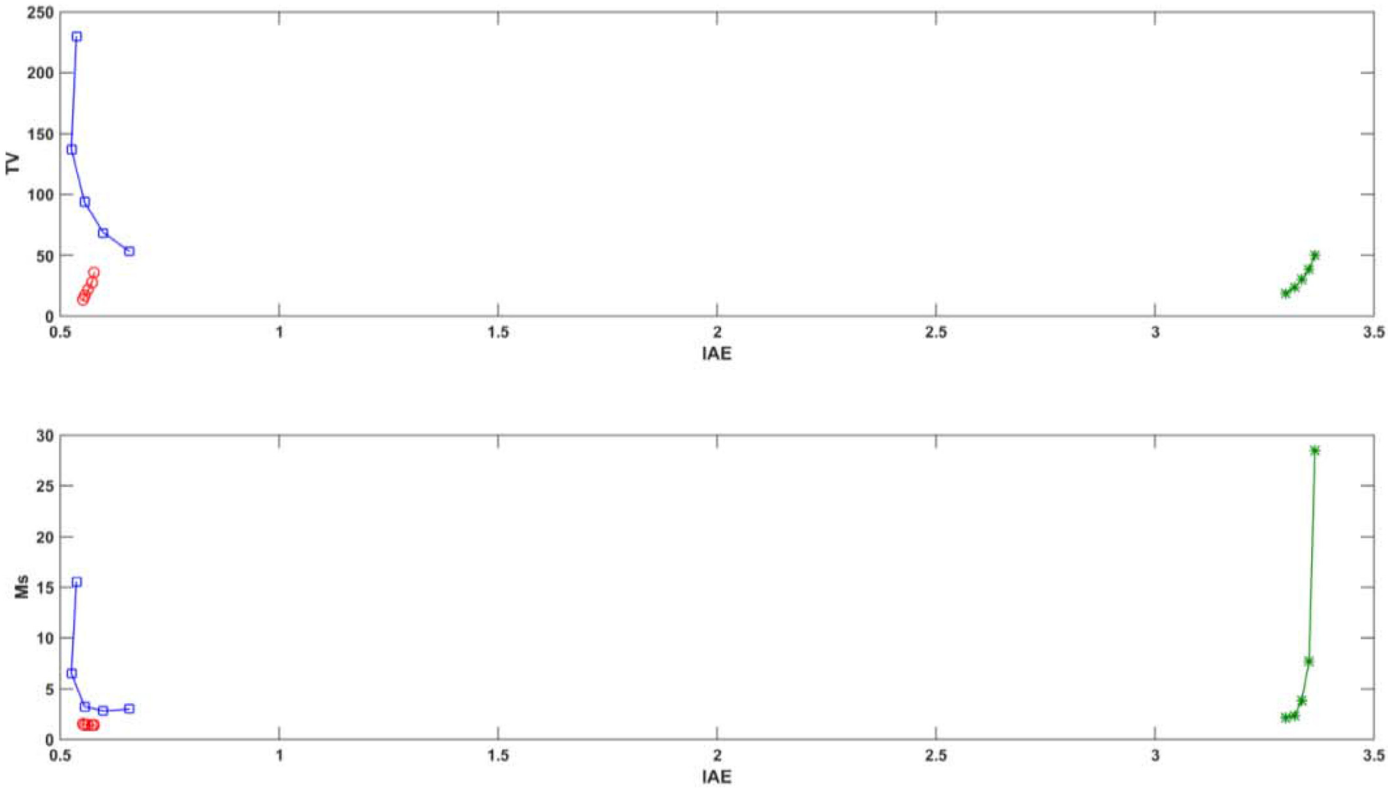

Fig. 9. FOPID controller performance for step set point changes of unit magnitude with L/T ratio of 0.3 for the three tuning methods.

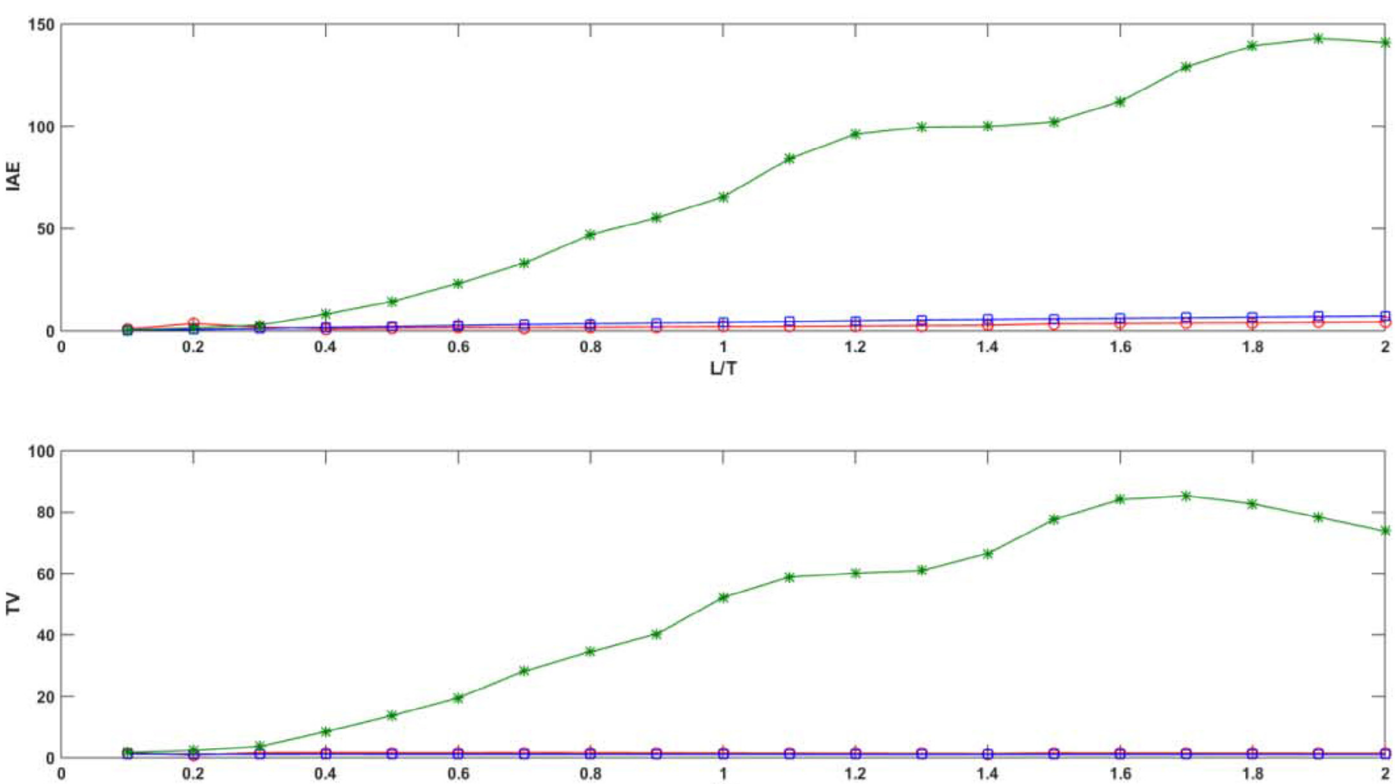

Fig. 10. Increasing L/T ratio versus performance metrics of FOPI controller for step disturbance changes of unit magnitude for the three tuning methods.

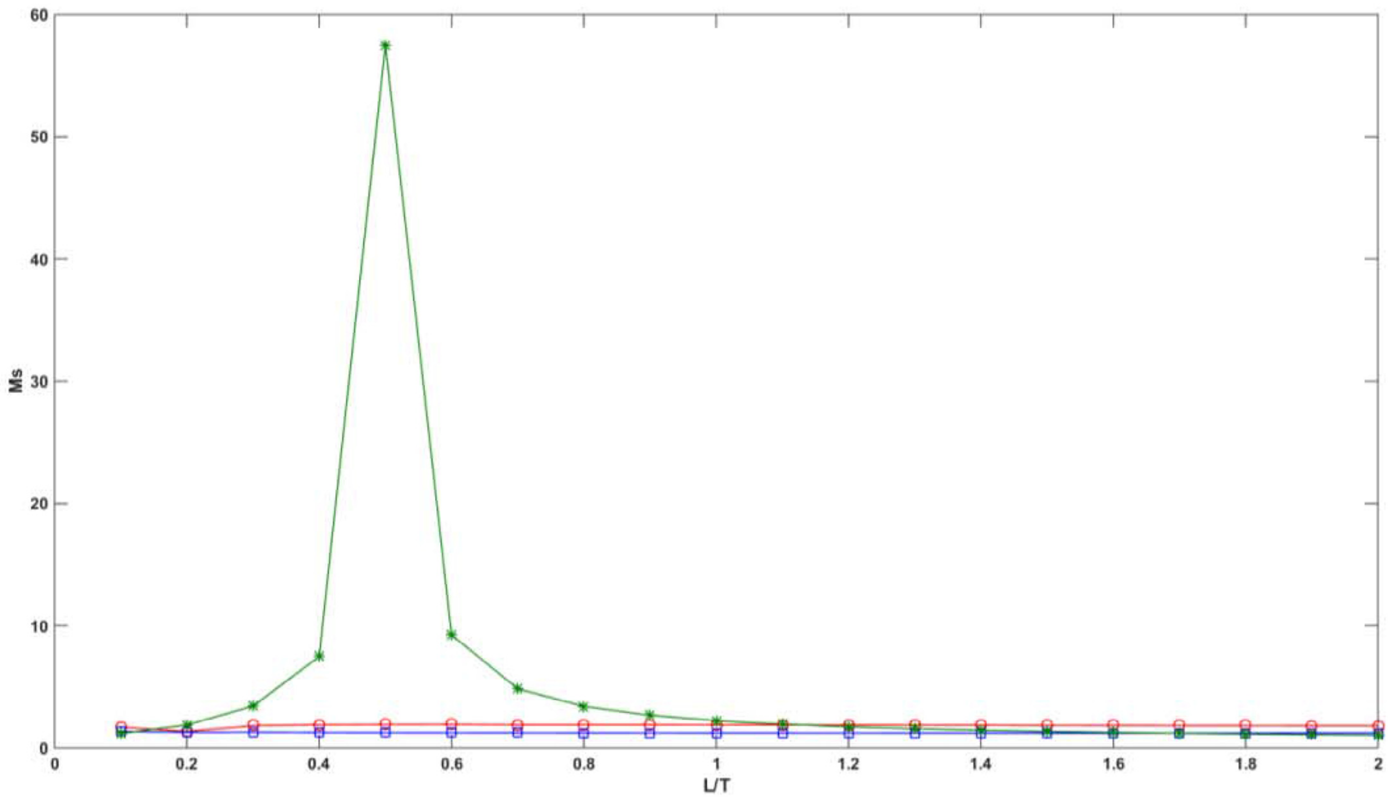

Fig. 11. Robustness of FOPI controller for step disturbance changes of unit magnitude with increasing L/T ratio for the three tuning methods.

for better closed loop performance except for a bit higher IAE values. Fig. 12 presents the performance of FOPI controller with the variation in tuning parameter λ. It ensures the overall performance of the closed loop system with the Gude et al. [9] method of controller settings.

From Fig. 13, it is observed that the Padula et al. [13] method gives lower values of IAE and TV followed by the Valerio et al. [11] method. Controller effort is almost the same in the case of all three methods. In terms of M_s value, the Padula et al. [13] method is giving a value of 1.4 throughout the scale of L/T ratio as it was designed to be. The Valerio et al. [11] method is giving M_s values beyond 2 when the process becomes delay significant. The Bayat [12] method is far from the requirements in terms of IAE and M_s. The result shown in Fig. 14 tells that FOPID controller is robust with the controller settings calculated using the Padula et al. [13] method. The performance of FOPID controller with the variation of λ illustrated in Fig. 15 clarifies that the Padula et al. [13] method is a better choice for robust performance of the controller and stable response of the FOPTD system.

5.3. Case study: liquid level system

It is easy to identify the optimal tuning rules of FOPI and FOPID controllers with the known value of L/T and with the recommendations given in Tables 3 and 4 for set point tracking and disturbance rejection problems. The optimal tuning rules identified are summarized in Tables 5 and 6.

Figures 16–23 illustrate the simulation results obtained for the level control system. To identify the effects of controllability index L/T, the plots of performance metrics IAE, TV and M_s with respect to increasing L/T ratio for FOPI controller tuning methods and FOPID controller tuning methods were illustrated for unit step type change in set point and disturbance. It is

Table 3
Recommendation of tuning rules for FOPI controller.

	Input	Lag dominant
Servo control	Unit step	1. Gude et al. [9] 2. Chen et al. [8]
Regulatory control	Unit step	1. Gude et al. [9] 2. Chen et al. [8]

Table 4
Recommendation of tuning rules for FOPID controller.

	Input	Lag dominant
Servo control	Unit step	1. Padula et al. [13]
Regulatory control	Unit step	1. Padula et al. [13] 2. Valerio et al. [11]

Table 5
Optimal FOPI tuning rules identified for level control system.

	Input	Lag dominant
Servo control	Unit step	1. Chen et al. [8] 2. Gude et al. [9]
Regulatory control	Unit step	1. Chen et al. [8] 2. Gude et al. [9]

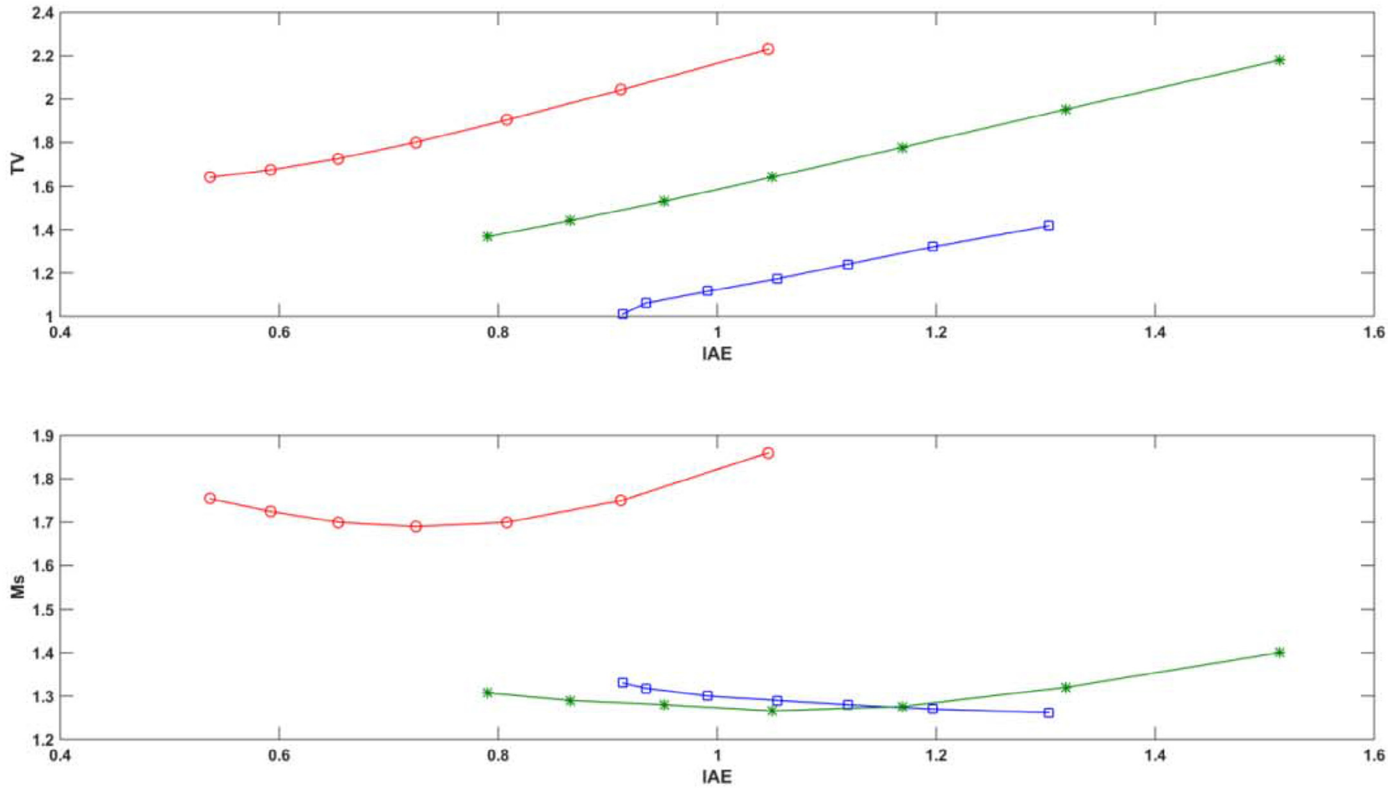

Fig. 12. FOPI controller performance for step disturbance changes of unit magnitude with L/T ratio of 0.3 for the three tuning methods.

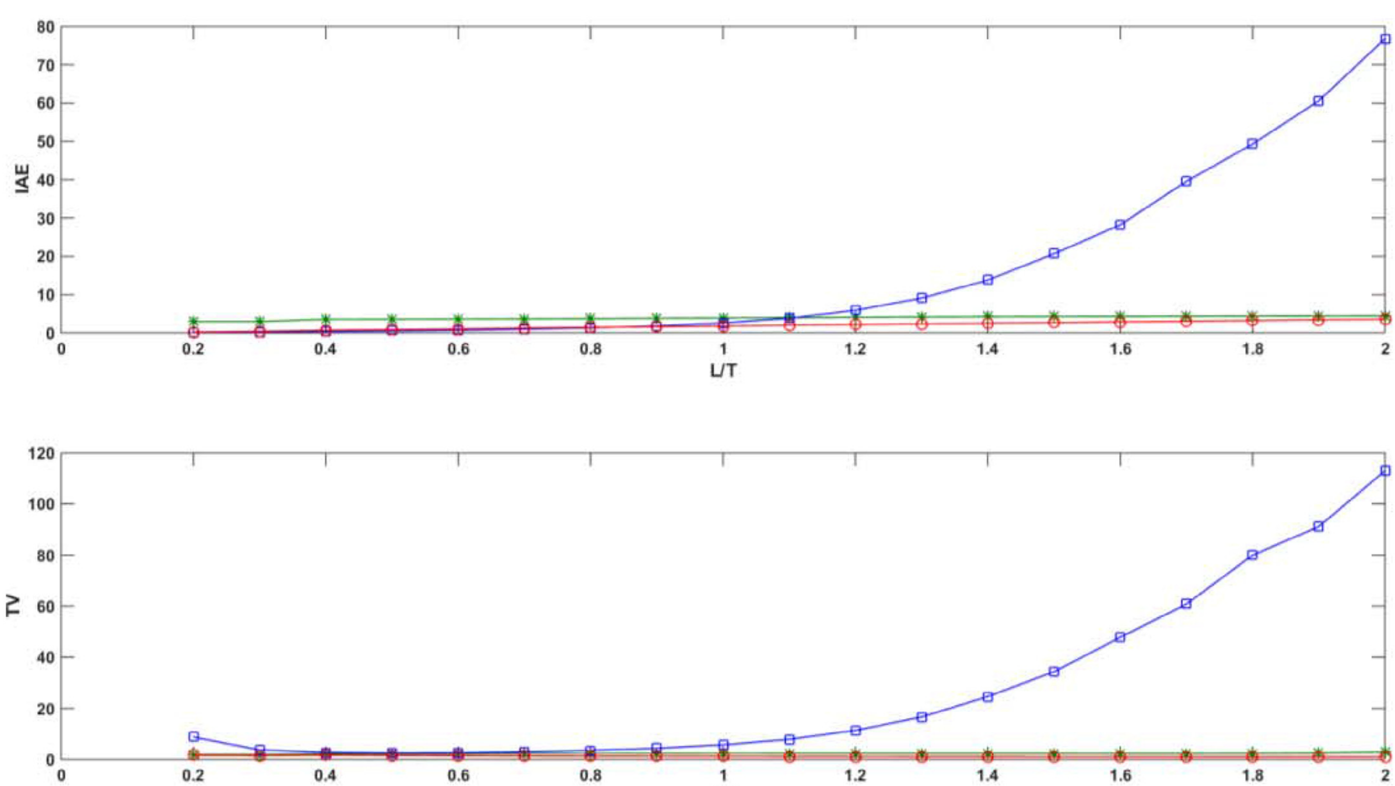

Fig. 13. Increasing L/T ratio versus performance metrics of FOPID controller for step disturbance changes of unit magnitude for the three tuning methods.

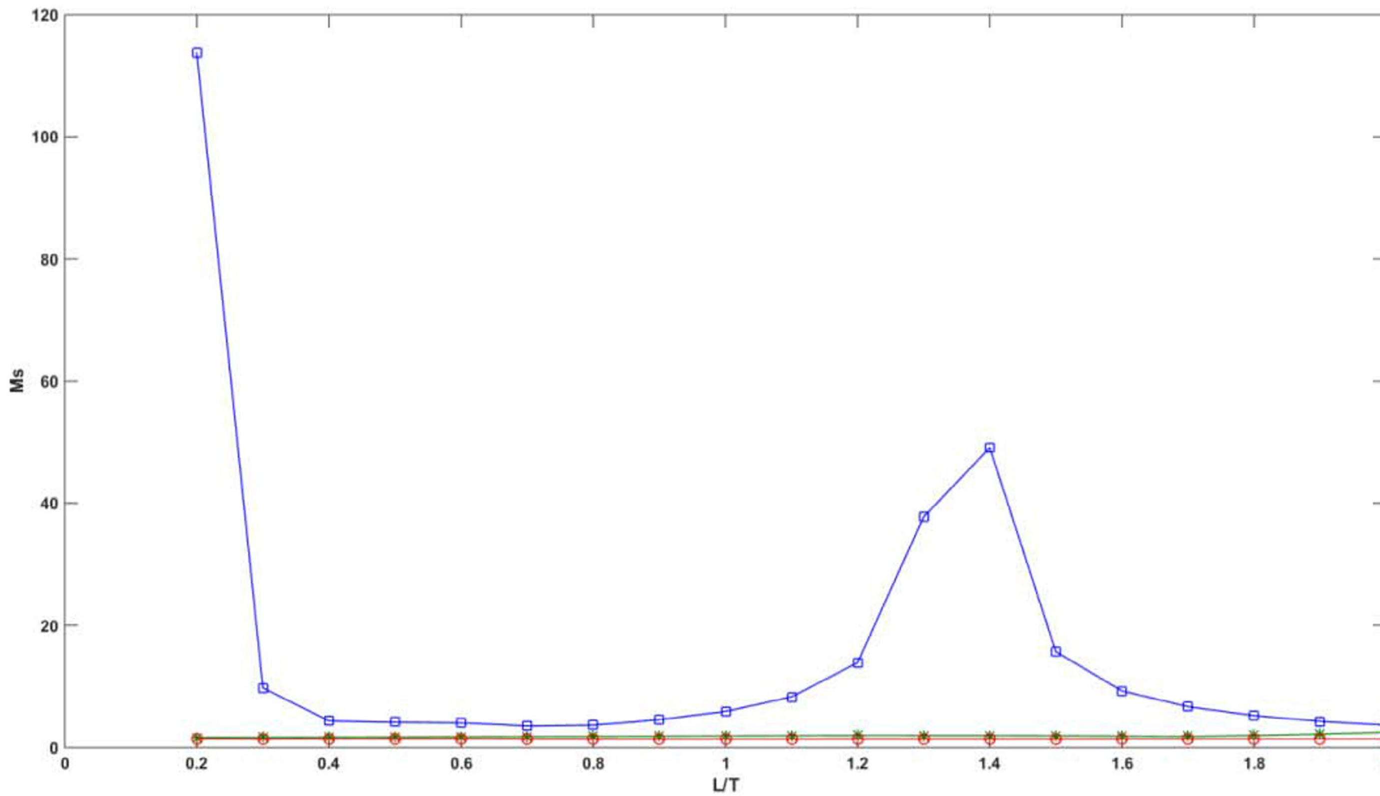

Fig. 14. Robustness of FOPID controller for step disturbance changes of unit magnitude with increasing L/T ratio for the three tuning methods.

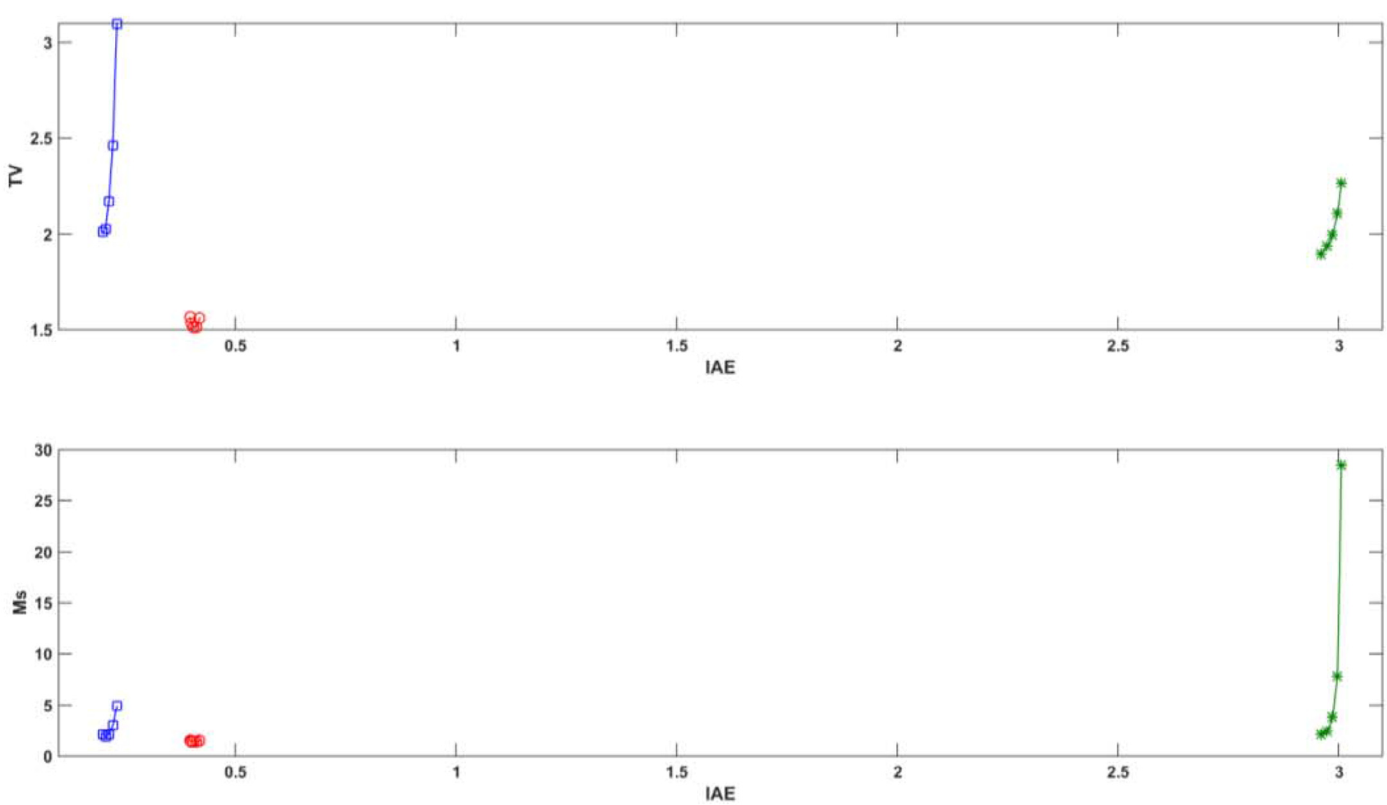

Fig. 15. FOPID controller performance for step disturbance changes of unit magnitude with L/T ratio of 0.3 for the three tuning methods.

Table 6
Optimal FOPID tuning rules identified for level control system.

	Inputs	Lag dominant
Servo control	Unit step	1. Padula et al. [13]
Regulatory control	Unit step	1. Padula et al. [13]

evident from Figs 16–17 and Figs 20–21 that the Chen et al. [8] method of tuning FOPI controller shows minimum values of IAE and M_s for servo control and regulatory control problems. The only limitation observed with the Chen et al. [8] method is that it produces an offset in the response if the fractional order λ is less than 1. The simulation results do not include the Bhambhani et al. [10] method of FOPI controller response. This is because the controller settings result in the negative value of tuning parameter λ which causes improper transfer function of the process model. It is apparent from Figs 18–19 and Figs 22–23 that the Padula et al. [13] method is the superior choice for tuning FOPID controller for servo control and regulatory control problems. The optimal tuning rules recommended in Tables 3 and 4 match with the results obtained which are shown in Tables 5 and 6. The Valerio et al. [11] method cannot be used for this case study as the process model parameters L and T are higher than the designed values.

Better closed loop response can be achieved when FOPI controller is tuned by the Chen et al. [8] and Gude et al. [9] tuning rules. But the application of Chen et al. [8] tuning rules for some lag dominant processes may result in an overshoot in the response. The Gude et al. [9] tuning rules are suggested for such processes. The high rise time, overshoot and settling time make the Bhambhani et al. [10] tuning rules least preferred for tuning the FOPI controller. The application of FOPID tuning rules by Valerio et al. [11] is limited to the processes with s-shaped step response. A further tuning is often required for the controller tuned by the Valerio et al. [11] tuning rules. The all time choice for tuning the FOPID controller is the Padula et al. [13] tuning method because the closed loop response had witnessed lower values of performance indices. The Bayat [12] tuning rules result in higher values of performance indices and hence not a choice for tuning the FOPID controller but reject disturbance effectively from some of the processes.

6. Conclusions

This paper recommends optimal $PI^{\lambda}/PI^{\lambda}D^{\mu}$ tuning rules for servo and regulatory control of processes in the industries when there is step change in the input variables (set point, disturbance) of process loop. For servo control problem, the Gude et al. [9] method is recommended followed by the Chen et al. [8] method. An offset occurs if the Chen et al. [8] method is used for servo control of lag dominant process if λ is less than 1. Hence, the Gude et al. [9] method is a primary choice for lag dominant processes. For more robust setting, one needs a compromise between the Chen et al. [8] method and the Gude et al. [9] method. The Padula et al. [13] $PI^{\lambda}D^{\mu}$ tuning method is recommended for servo control problem of all types of industrial processes. For regulatory control, the Chen et al. [8] method and the Gude et al. [9] method can alternatively be used with a line between IAE and M_s. In case of $PI^{\lambda}D^{\mu}$ tuning

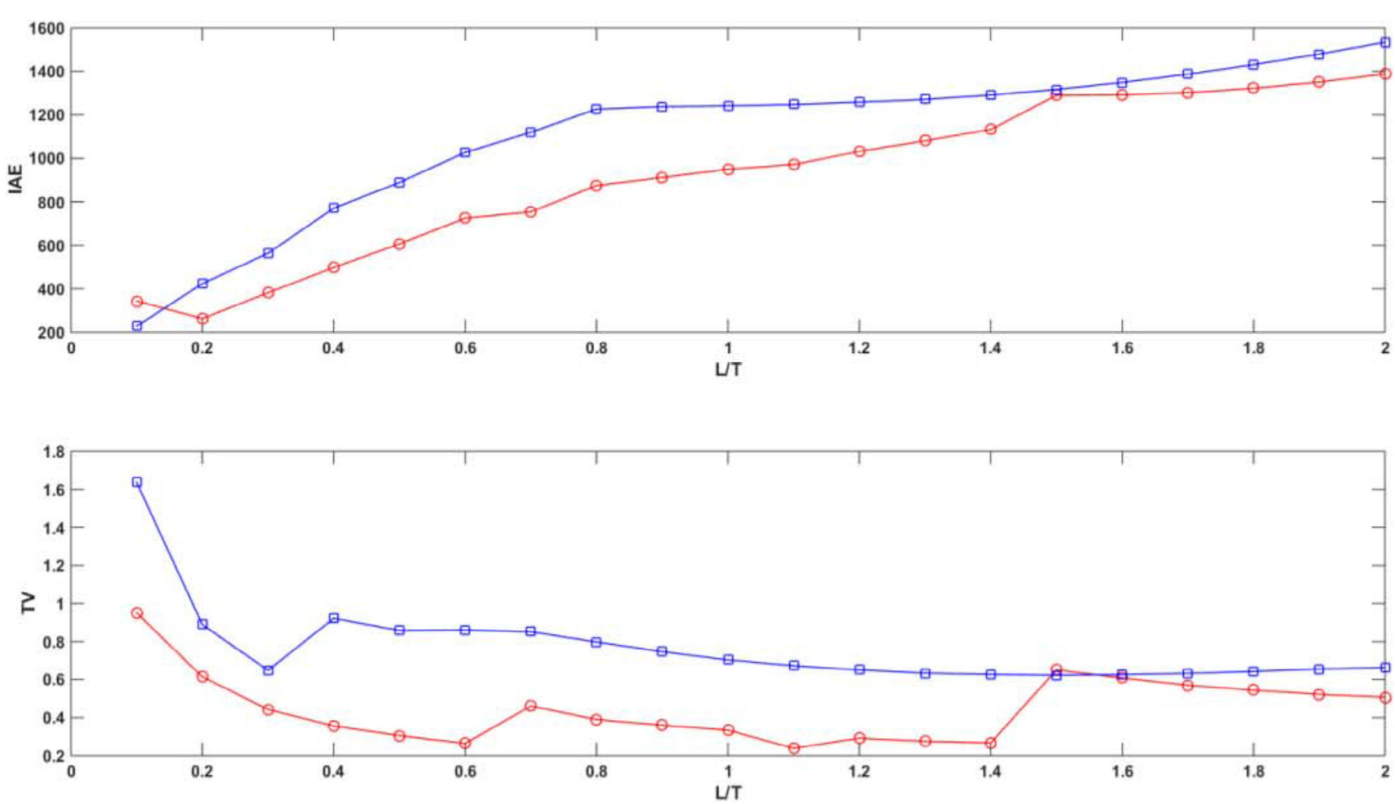

Fig. 16. Increasing L/T ratio versus performance metrics of FOPI controller for step set point changes of unit magnitude for the case study.

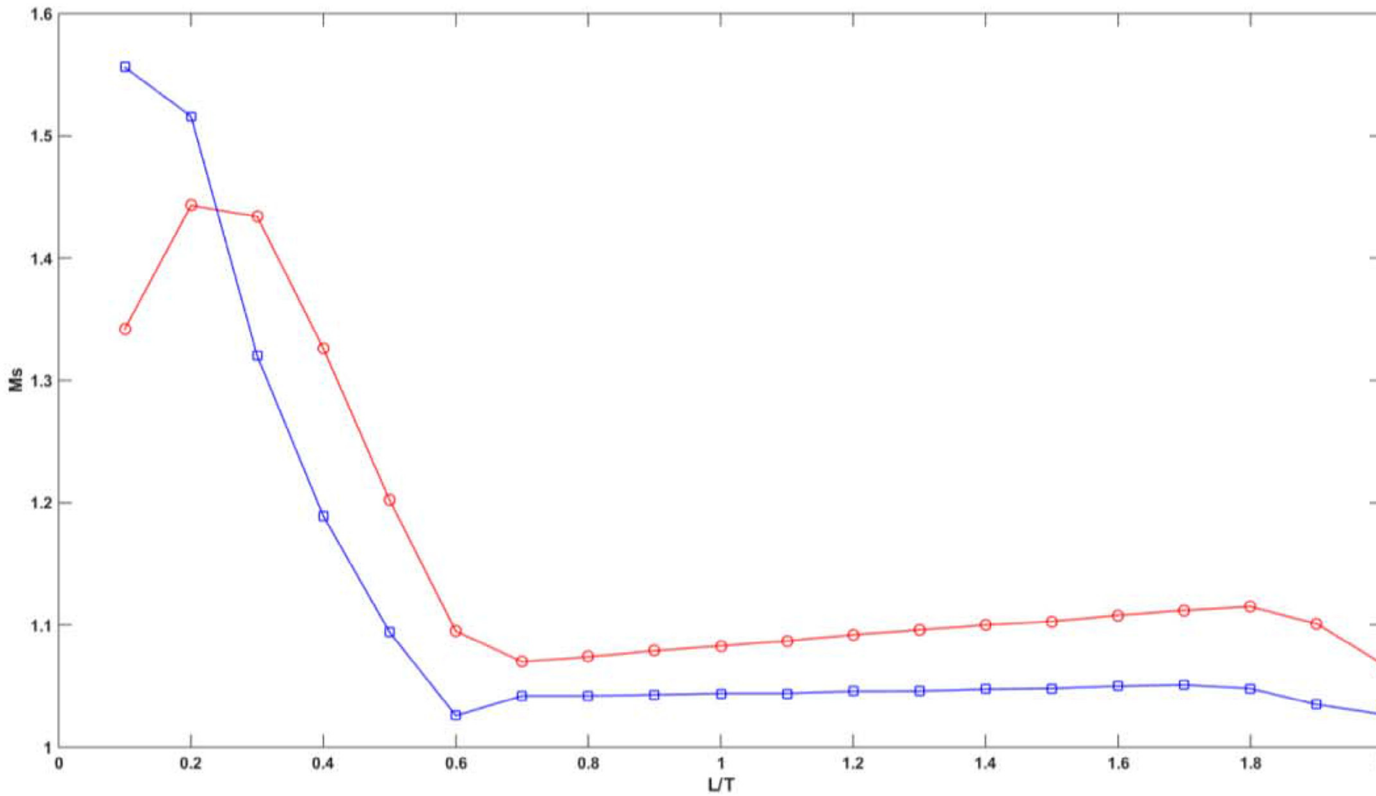

Fig. 17. Robustness of FOPI controller for step set point changes of unit magnitude with increasing L/T ratio for the case study.

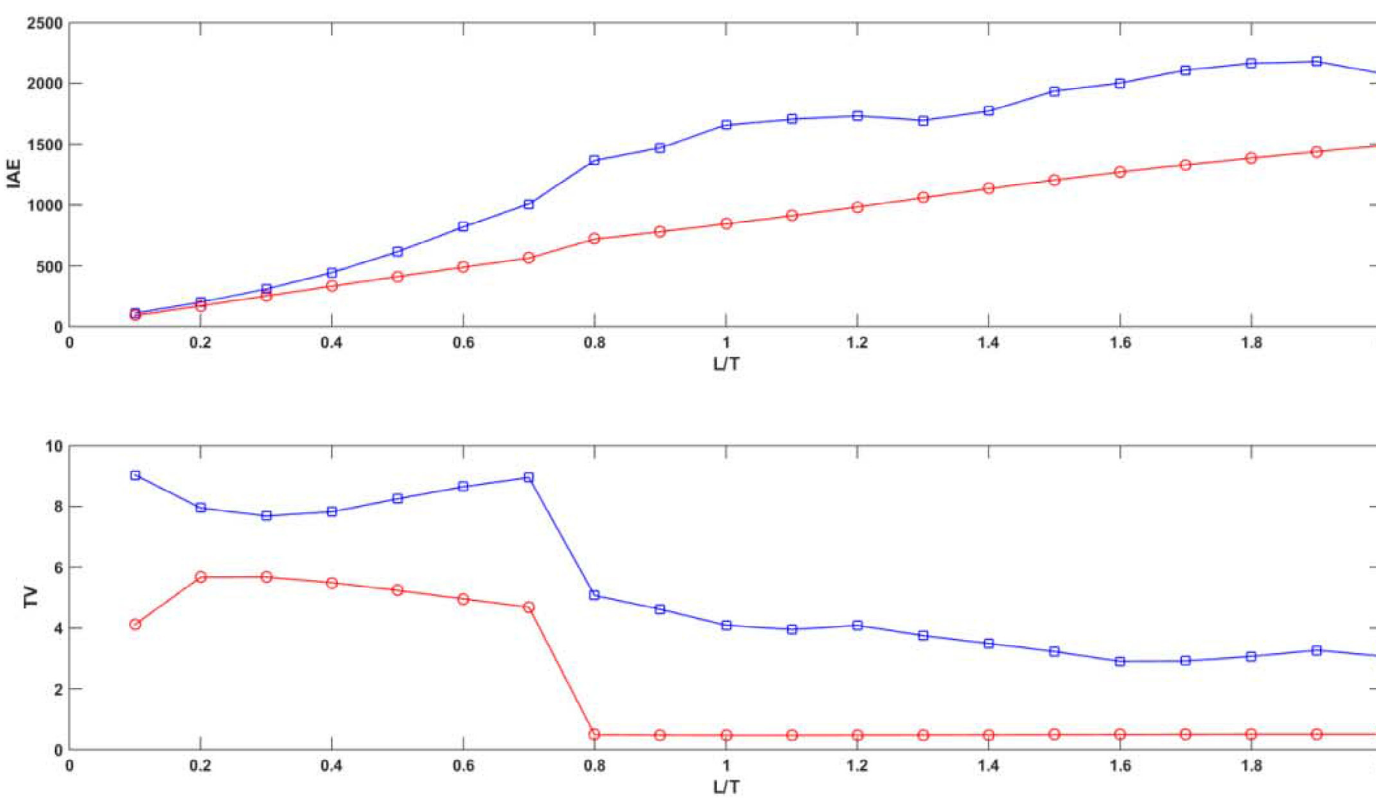

Fig. 18. Increasing L/T ratio versus performance metrics of FOPID controller for step set point changes of unit magnitude for the case study.

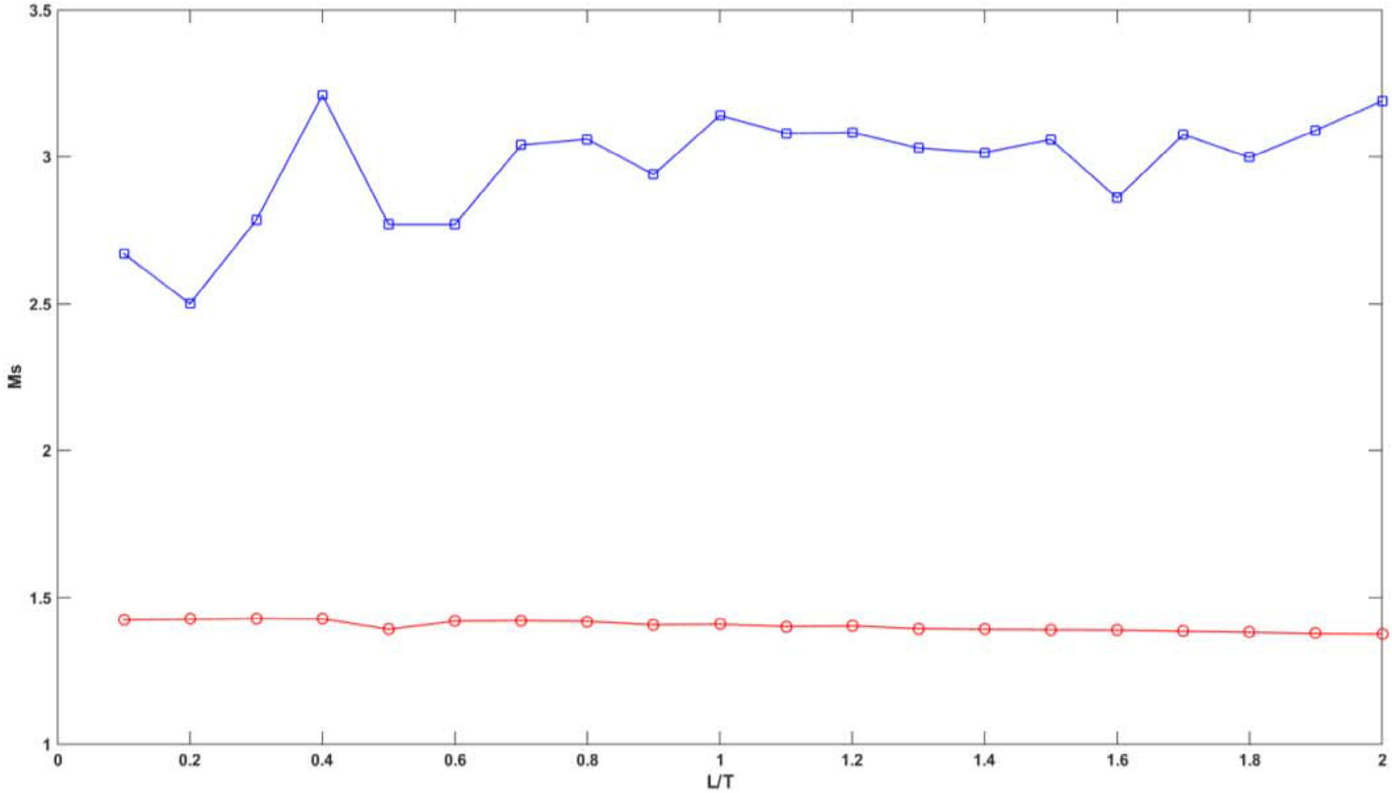

Fig. 19. Robustness of FOPID controller for step set point changes of unit magnitude with increasing L/T ratio for the case study.

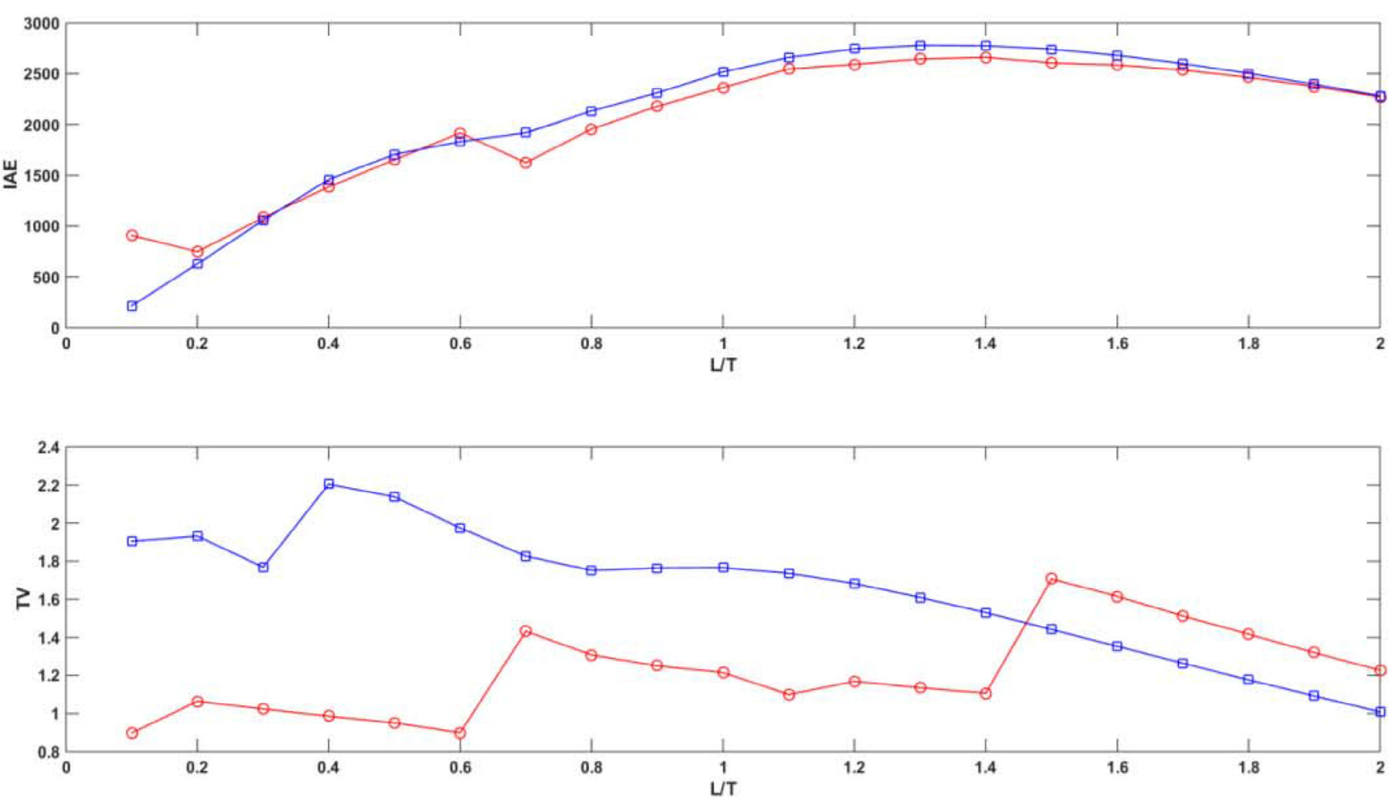

Fig. 20. Increasing L/T ratio versus performance metrics of FOPI controller for step disturbance changes of unit magnitude for the case study.

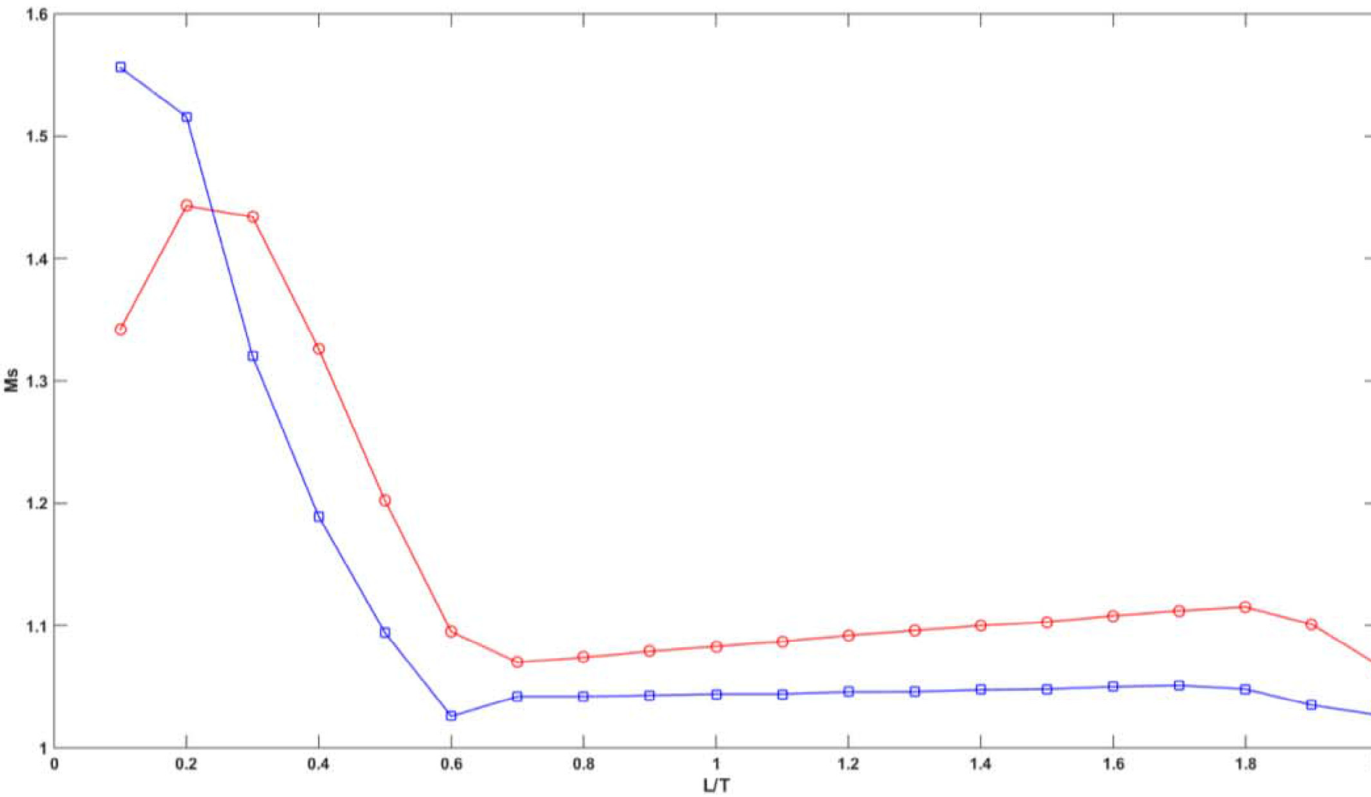

Fig. 21. Robustness of FOPI controller for step disturbance changes of unit magnitude with increasing L/T ratio for the case study.

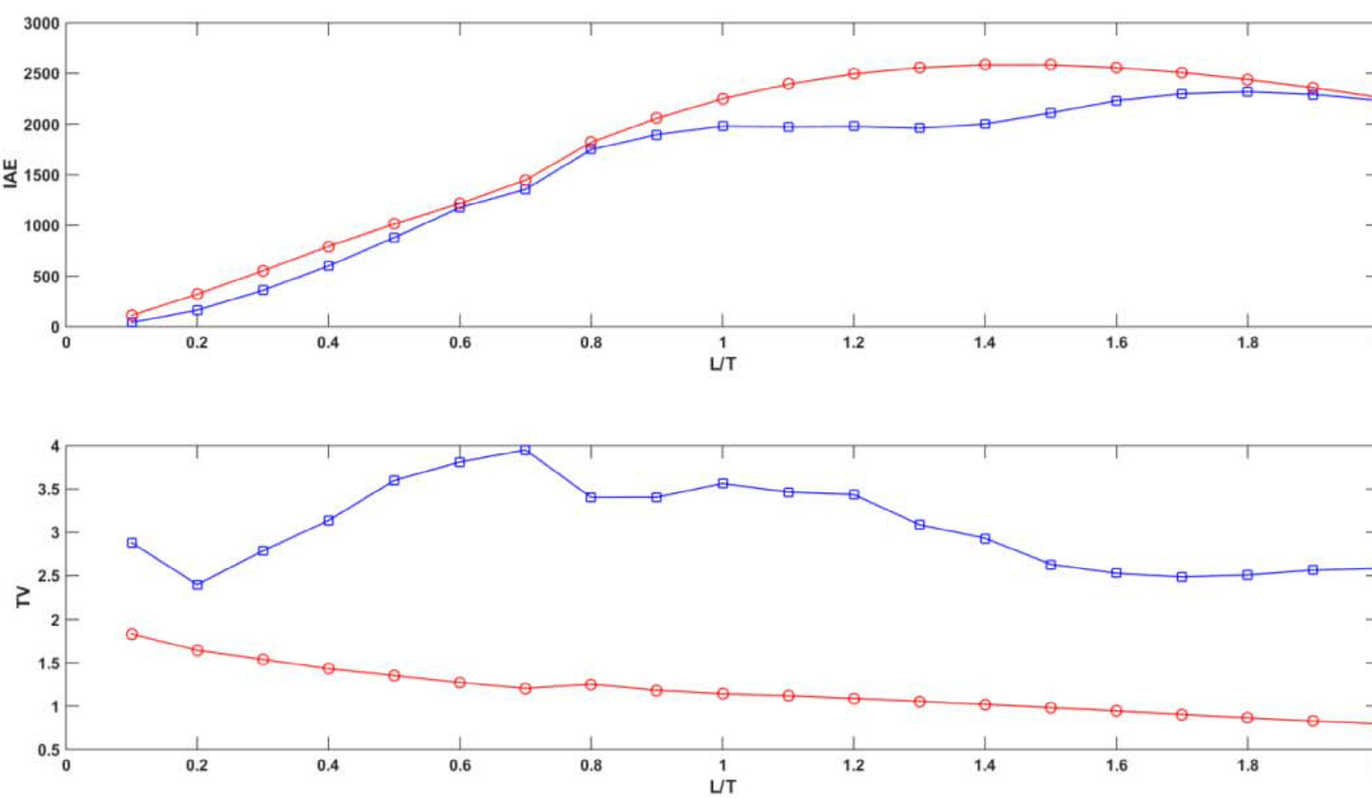

Fig. 22. Increasing L/T ratio versus performance metrics of FOPID controller for step disturbance changes of unit magnitude for the case study.

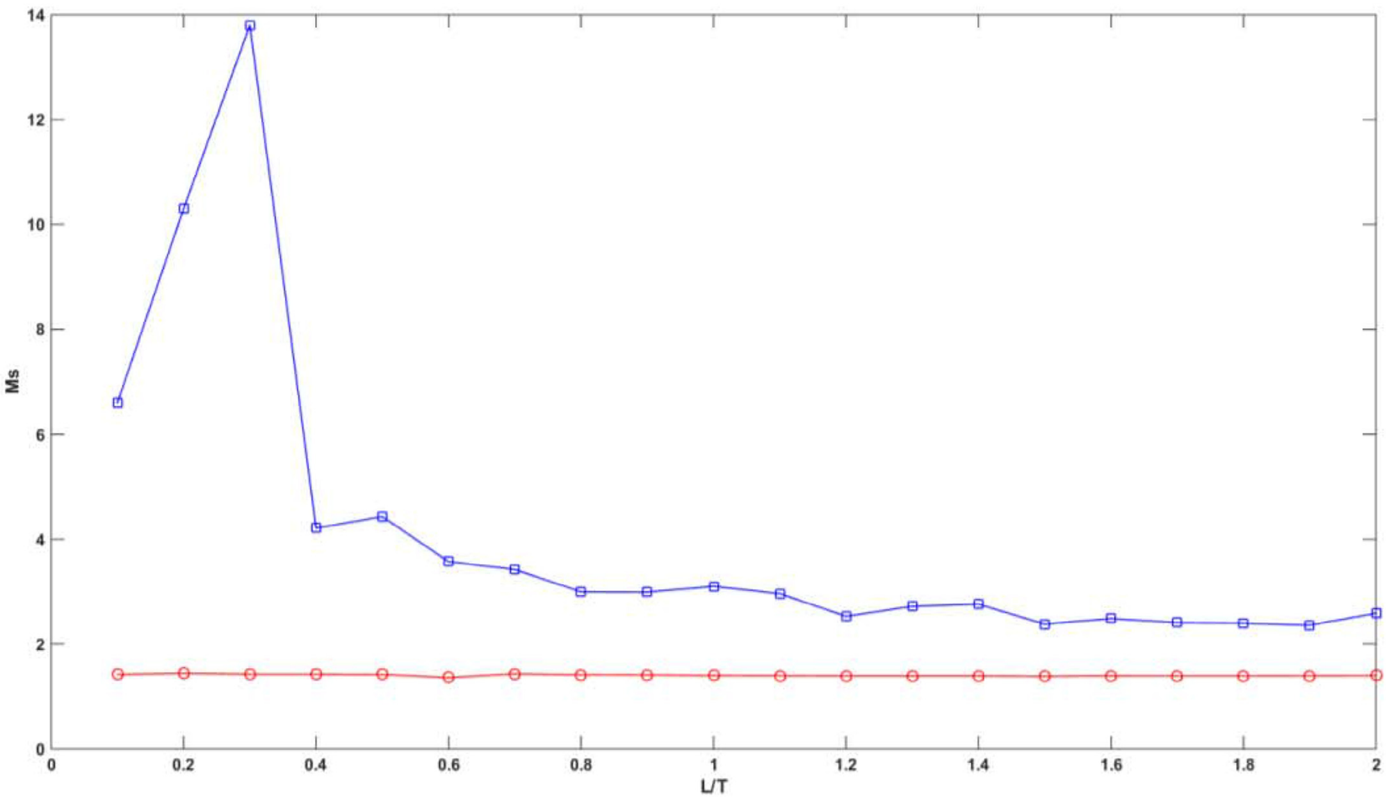

Fig. 23. Robustness of FOPID controller for step disturbance changes of unit magnitude with increasing L/T ratio for the case study.

methods, the Padula et al. [13] method is recommended for fast response and robustness of all kinds of industrial processes.

References

[1] I. Podlubny, Fractional-order systems and $PI^{\lambda}D^{\mu}$ controllers, IEEE Trans. Automat. Contr. 44 (1) (1999) 208–214.

[2] P. Shah, S. Agashe, Review of fractional PID controller, Mechatronics 38 (2016) 29–41.

[3] Y. Luo, Y.Q. Chen, Fractional Order Motion Control, first ed., John Wiley and Sons, Chichester, UK, 2013.

[4] R.D. Keyser, C.I. Muresan, C.M. Ionescu, A novel auto-tuning method for fractional order PI/PD controllers, ISA Trans. 62 (2016) 268–275.

[5] A.A. Kesarkar, N. Selvaganesan, Tuning of optimal fractional order PID controller using an artificial bee colony algorithm, Syst. Sci. Control Eng. 3 (2015) 99–105.

[6] F. Padula, A. Visioli, On the fragility of fractional-order PID controllers for FOPDT processes, ISA Trans. 60 (2016) 228–243.

[7] D.E. Seborg, T.F. Edgar, D.A. Mellichamp, Process Dynamics and Control, second ed., John Wiley and Sons, New York, 2004.

[8] Y. Chen, T. Bhaskaran, D. Xue, Practical tuning rule development for fractional order proportional and integral controllers, J. Comput. Nonlinear Dyn. 3 (2) (2008) 214031–214038.

[9] J.J. Gude, E. Kahoraho, Simple tuning rules for fractional PI controllers. IEEE Conference on Emerging Technologies & Factory Automation, (2009) 4910–4915.

[10] V. Bhambhani, Y.Q. Chen, D. Xue, Optimal Fractional Order Proportional Integral Controller for Varying Time-Delay Systems. 17th IFAC World Congress, Seoul, Korea, July 6–11 (2008).

[11] D. Valerio, J.S.D. Costa, Tuning of fractional PID controllers with Ziegler–Nichols-type rules, Signal Processing 86 (2006) 2771–2784.

[12] F.M. Bayat, General rules for optimal tuning the $PI^{\lambda}D^{\mu}$ controllers with application to first-order plus time delay processes, Can. J. Chem. Eng. 9999 (2012) 1–11.

[13] F. Padula, A. Visioli, Tuning rules for optimal PID and fractional order PID controllers, J. Process Control 21 (2011) 69–81.

[14] D. Chen, D.E. Seborg, PI/PID controller design based on direct synthesis and disturbance rejection, Ind. Eng. Chem. Res. 41 (2002) 4807–4822.

[15] C.A. Monje, B.M. Vinagre, V. Feliu, Y.Q. Chen, Tuning and auto-tuning of fractional order controllers for industry applications, Control Eng. Pract. 16 (2008) 798–812.